

Thinking Recursively

Part II

Outline for Today

- ***The Recursive Leap of Faith***
 - On trusting the contract.
- ***Enumerating Subsets***
 - A classic combinatorial problem.
- ***Decision Trees***
 - Generating all solutions to a problem.

Some Quick Refreshers

Set Refresher

- What's printed at Line *A* and Line *B*?

```
Set<int> mySet = {1, 2, 3};
```

```
cout << (mySet + 4) << endl; // Line A
```

```
cout << (mySet - 3) << endl; // Line B
```

Set Refresher

- What's printed at Line *A* and Line *B*?

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Set<int> mySet = {1, 2, 3};
```

```
cout << (mySet + 4) << endl; // Line A
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Set Refresher

- What's printed at Line *A* and Line *B*?

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```

```
cout << (mySet + 4) << endl; // Line A
```

```
cout << (mySet - 3) << endl; // Line B
```

```
{1, 2, 3}
```

```
Set<int> mySet
```

Set Refresher

- What's printed at Line *A* and Line *B*?

```
Set<int> mySet = {1, 2, 3};
```

```
cout << (mySet + 4) << endl; // Line A
```

```
cout << (mySet - 3) << endl; // Line B
```

{1, 2, 3}

```
Set<int> mySet
```

Set Refresher

- What's printed at Line *A* and Line *B*?

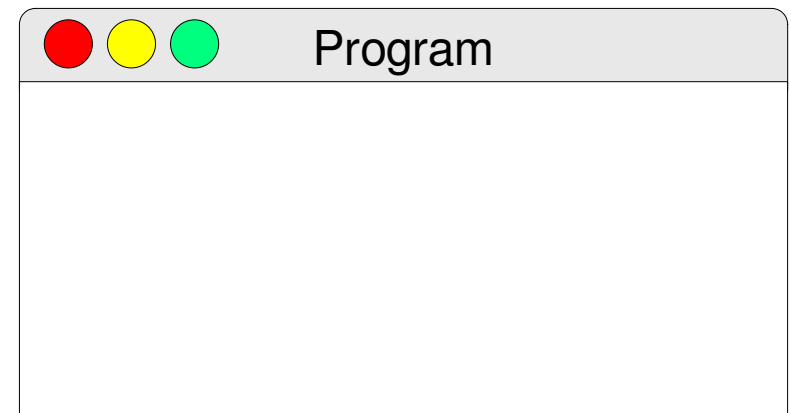
```
Set<int> mySet = {1, 2, 3};
```

```
cout << (mySet + 4) << endl; // Line A
```

```
cout << (mySet - 3) << endl; // Line B
```

{1, 2, 3}

```
Set<int> mySet
```



Set Refresher

- What's printed at Line *A* and Line *B*?

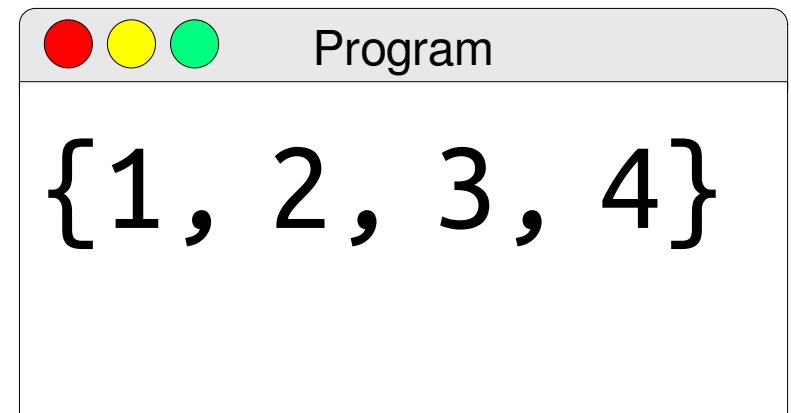
```
Set<int> mySet = {1, 2, 3};
```

```
cout << (mySet + 4) << endl; // Line A
```

```
cout << (mySet - 3) << endl; // Line B
```

{1, 2, 3}

```
Set<int> mySet
```



Set Refresher

- What's printed at Line *A* and Line *B*?

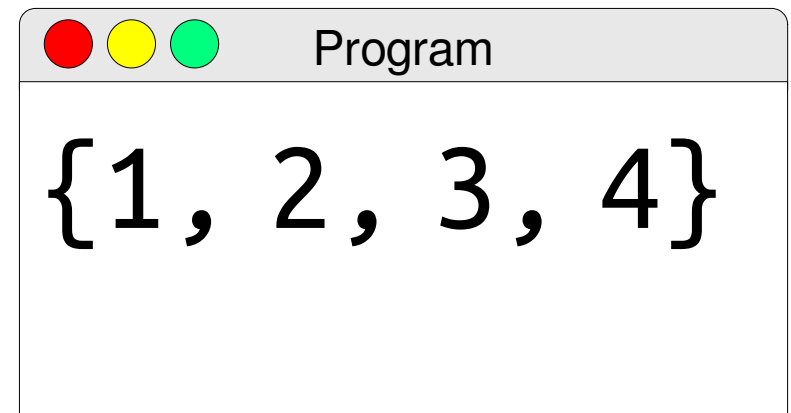
```
Set<int> mySet = {1, 2, 3};
```

```
cout << (mySet + 4) << endl; // Line A
```

```
cout << (mySet - 3) << endl; // Line B
```

{1, 2, 3}

```
Set<int> mySet
```



Set Refresher

- What's printed at Line *A* and Line *B*?

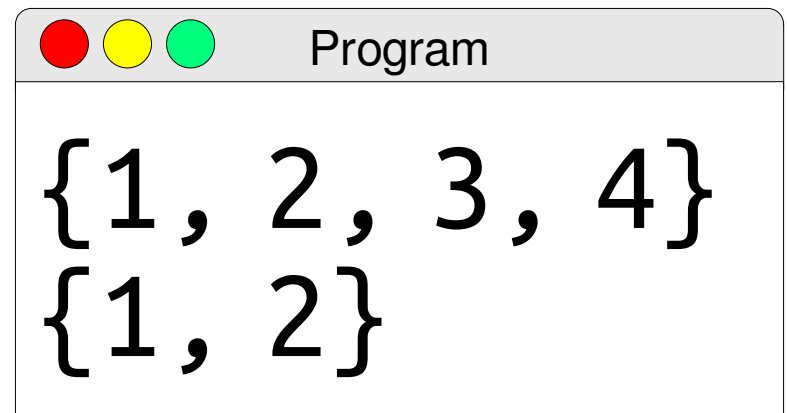
```
Set<int> mySet = {1, 2, 3};
```

```
cout << (mySet + 4) << endl; // Line A
```

```
cout << (mySet - 3) << endl; // Line B
```

{1, 2, 3}

```
Set<int> mySet
```



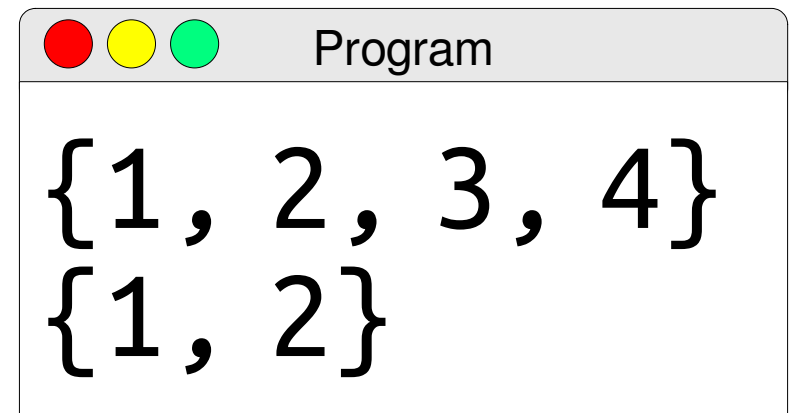
Set Refresher

- What's printed at Line *A* and Line *B*?

```
Set<int> mySet = {1, 2, 3};  
cout << (mySet + 4) << endl; // Line A  
cout << (mySet - 3) << endl; // Line B
```

{1, 2, 3}

Set<int> mySet



Recursion Refresher

- What does this code print?

```
void squigglebah(int n) {  
    if (n != 0) {  
        squigglebah(n - 1);  
        cout << n << endl;  
    }  
}  
  
squigglebah(2);
```

```
squiggalebah(2);
```

Squigglebah(2)

```
void squigglebah(int n) {  
    if (n != 0) {  
        squigglebah(n - 1);  
        cout << n << endl;  
    }  
}
```

2

int n

5 squigglebah(2)

```
void squigglebah(int n) {  
    if (n != 0) {  
        squigglebah(n - 1);  
        cout << n << endl;  
    }  
}
```

2

int n

Squigglebah(2)

```
void squigglebah(int n) {  
    if (n != 0) {  
        squigglebah(n - 1);  
        cout << n << endl;  
    }  
}
```

2

int n

void squigglebah(int n) {

void squigglebah(int n) {

2

void squigglebah(int n) {

if (n != 0) {

squigglebah(n - 1);

cout << n << endl;

}

}

1

int n

void squigglebah(int n) {

void squigglebah(int n) {

2

void squigglebah(int n) {

if (n != 0) {

1

squigglebah(n - 1);

int n

cout << n << endl;

}

}

void squigglebah(int n) {

void squigglebah(int n) {

2

void squigglebah(int n) {

if (n != 0) {

squigglebah(n - 1);

cout << n << endl;

1

int n

}

void squigglebah(int n) {

void squigglebah(int n) {

2

void squigglebah(int n) {

1

void squigglebah(int n) {

if (n != 0) {

squigglebah(n - 1);

cout << n << endl;

}

}

0

int n

void squigglebah(int n) {

void squigglebah(int n) {

2

void squigglebah(int n) {

1

void squigglebah(int n) {

if (n != 0) {

0

squigglebah(n - 1);

int n

cout << n << endl;

}

}

void squigglebah(int n) {

void squigglebah(int n) {

2

void squigglebah(int n) {

1

void squigglebah(int n) {

if (n != 0) {

squigglebah(n - 1);

cout << n << endl;

}

}

0

int n

void squigglebah(int n) {

void squigglebah(int n) {

2

void squigglebah(int n) {

if (n != 0) {

squigglebah(n - 1);

cout << n << endl;

1

int n

}

void squigglebah(int n) {

void squigglebah(int n) {

2

void squigglebah(int n) {

if (n != 0) {

squigglebah(n - 1);

cout << n << endl;

1

int n

void squigglebah(int n) {

void squigglebah(int n) {

2

void squigglebah(int n) {

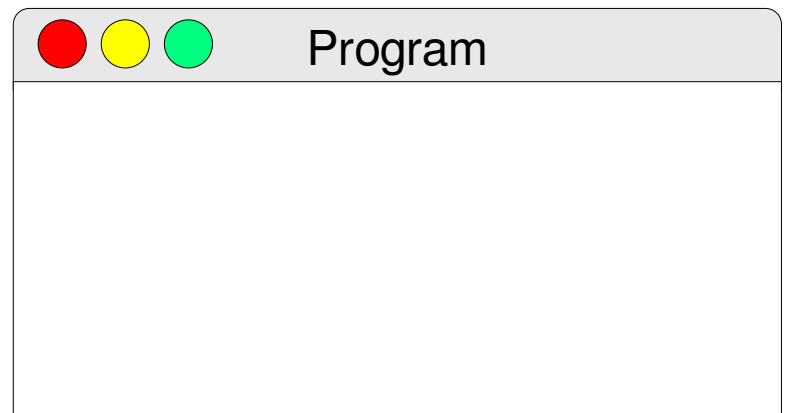
if (n != 0) {

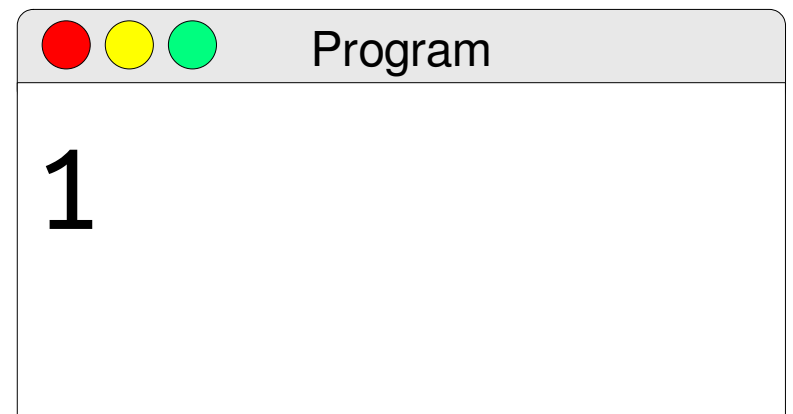
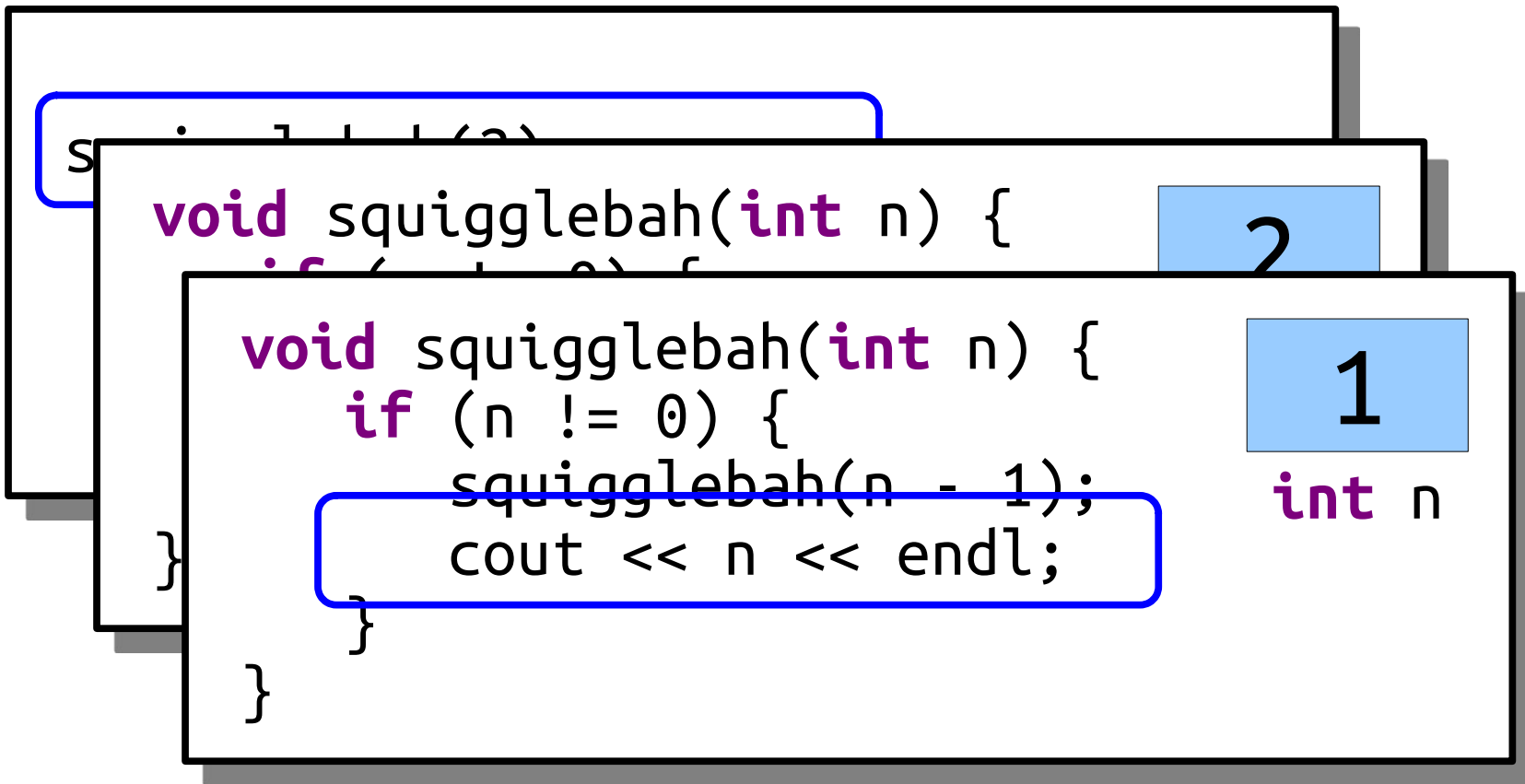
squigglebah(n - 1);

cout << n << endl;

1

int n





void squigglebah(int n) {

void squigglebah(int n) {

2

void squigglebah(int n) {

if (n != 0) {

squigglebah(n - 1);

cout << n << endl;

}

}

1

int n



Program

1

Squigglebah(2)

```
void squigglebah(int n) {  
    if (n != 0) {  
        squigglebah(n - 1);  
        cout << n << endl;  
    }  
}
```

2

int n



Program

1

Squigglebah(2)

```
void squigglebah(int n) {  
    if (n != 0) {  
        squigglebah(n - 1);  
        cout << n << endl;  
    }  
}
```

2

int n



Program

1

Squigglebah(2)

```
void squigglebah(int n) {  
    if (n != 0) {  
        squigglebah(n - 1);  
        cout << n << endl;  
    }  
}
```

2

int n



Program

1
2

Squigglebah(2)

```
void squigglebah(int n) {  
    if (n != 0) {  
        squigglebah(n - 1);  
        cout << n << endl;  
    }  
}
```

2

int n



Program

1

2

```
squiggalebah(2);
```



Program

1
2

```
squiggalebah(2);
```

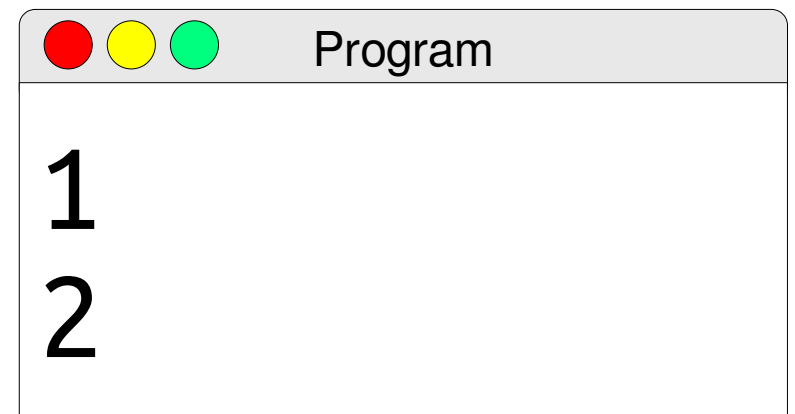


Program

1
2

```
squigglebah(2);
```

```
void squigglebah(int n) {  
    if (n != 0) {  
        squigglebah(n - 1);  
        cout << n << endl;  
    }  
}
```



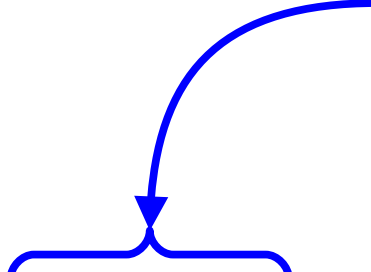
The Recursive Leap of Faith

The Contract

```
bool isVowel(char ch);
```

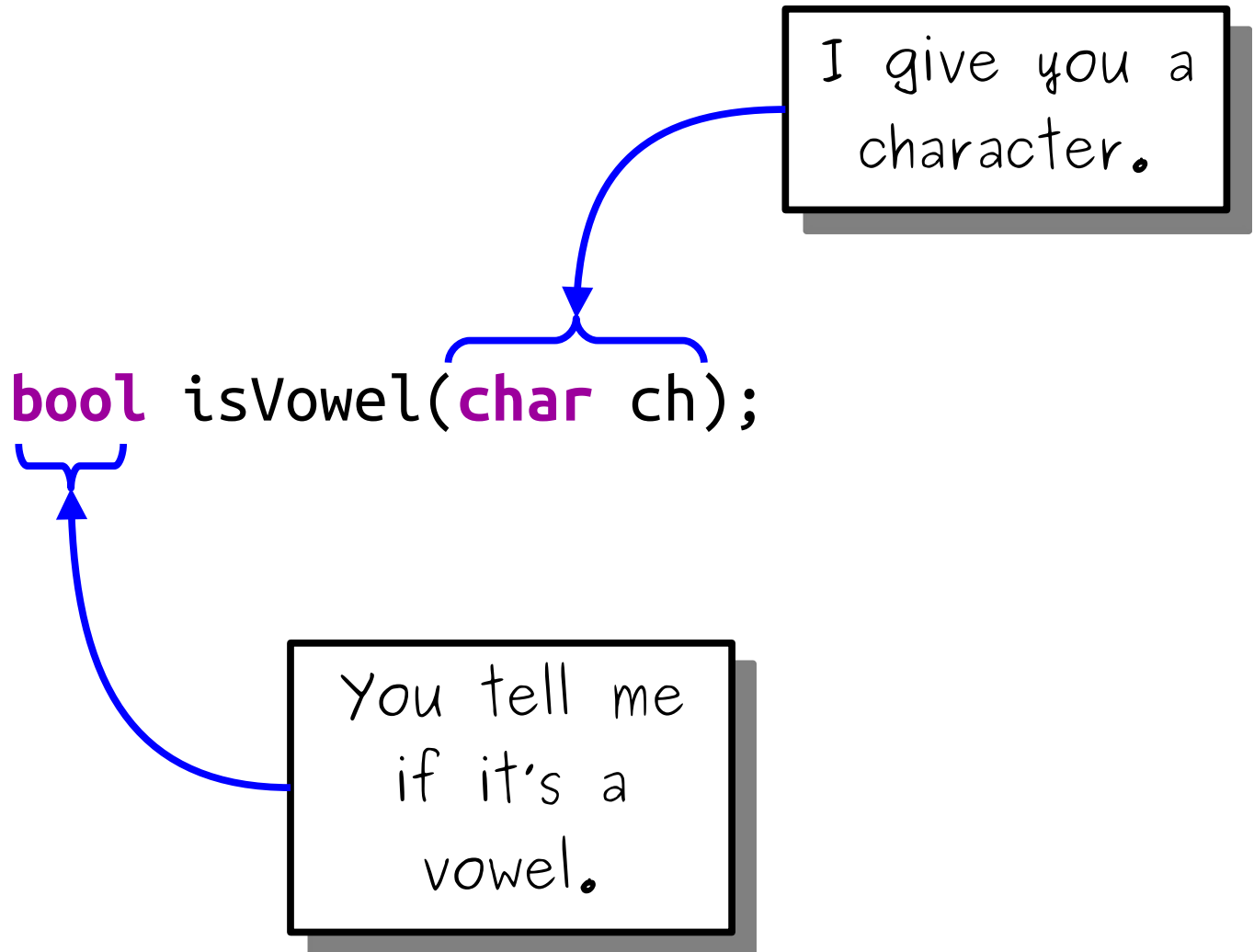
The Contract

I give you a
character.



```
bool isVowel(char ch);
```

The Contract



The Contract

```
bool isVowel(char ch) {  
    ch = toLowerCase(ch);  
    return ch == 'a' ||  
           ch == 'e' ||  
           ch == 'i' ||  
           ch == 'o' ||  
           ch == 'u';  
}
```

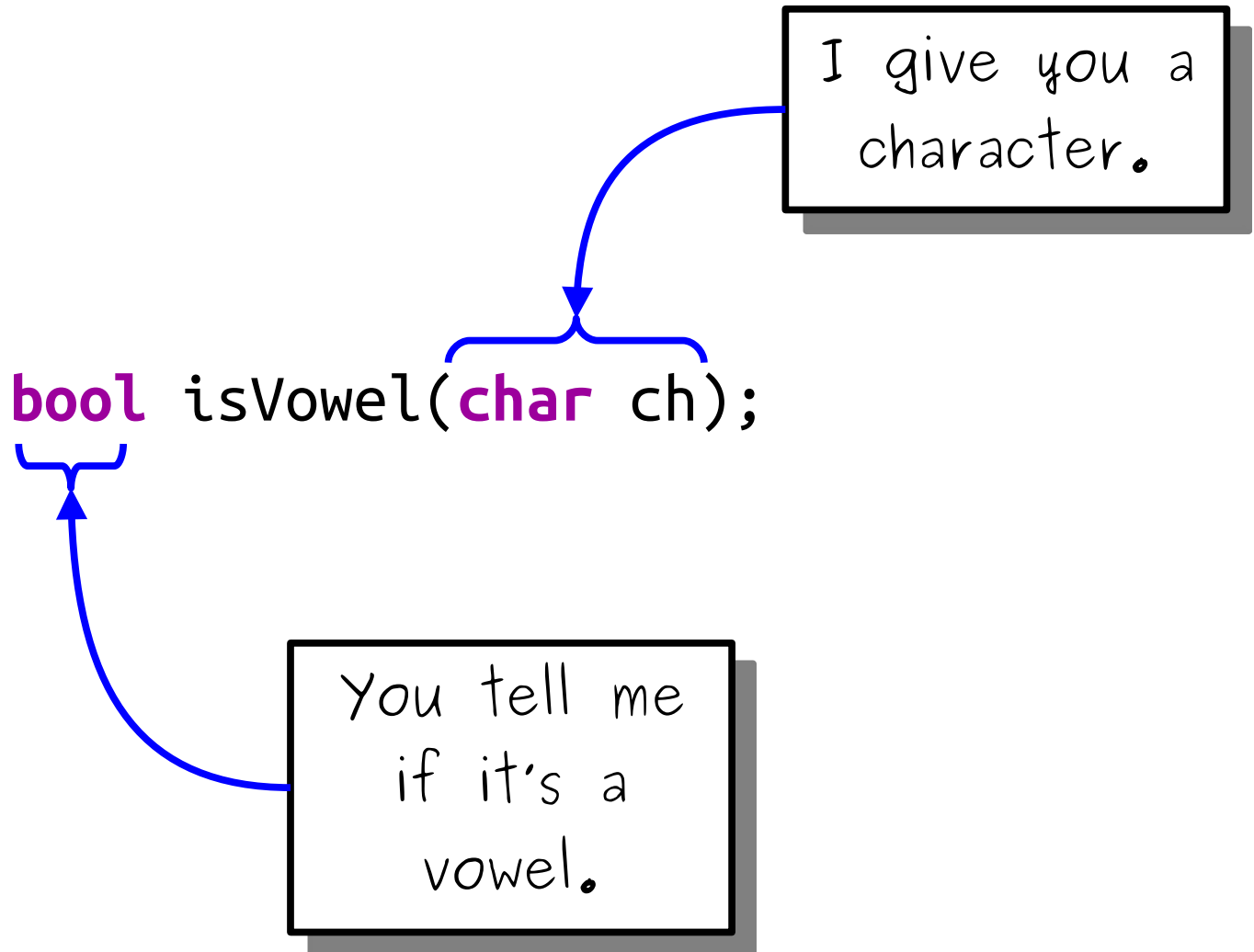
The Contract

```
bool isVowel(char ch) {  
    switch(ch) {  
        case 'A': case 'a':  
        case 'E': case 'e':  
        case 'I': case 'i':  
        case 'O': case 'o':  
        case 'U': case 'u':  
            return true;  
        default:  
            return false;  
    }  
}
```

The Contract

```
bool isVowel(char ch) {  
    ch = tolower(ch);  
    return string("aeiou").find(ch) != string::npos;  
}
```

The Contract



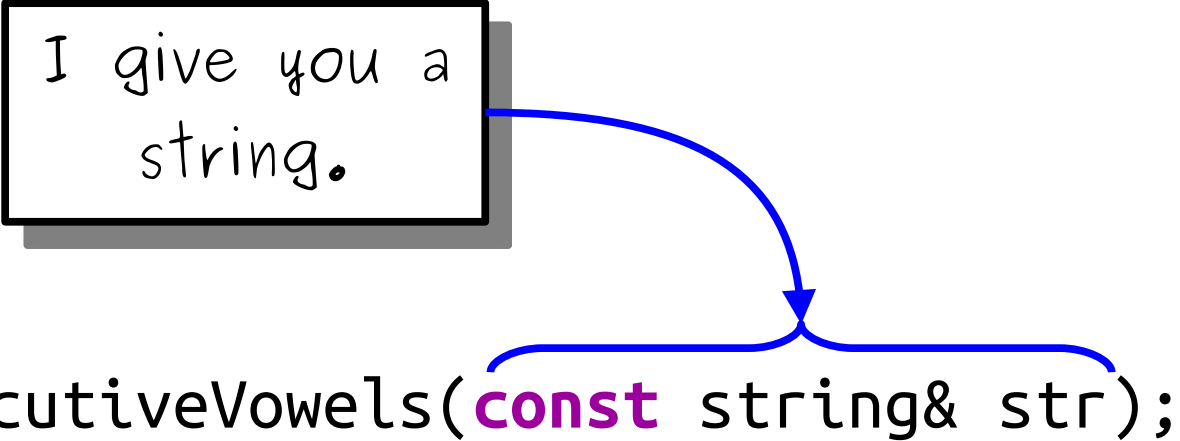
The Contract

The Contract

```
bool hasConsecutiveVowels(const string& str);
```

The Contract

I give you a
string.

A diagram illustrating a contract. A rectangular box with a black border and a grey drop shadow contains the handwritten text "I give you a string.". A blue arrow originates from the right side of this box and points to a curly brace in the code below. The curly brace is positioned under the parameter "const string& str" in the function signature of the code.

```
bool hasConsecutiveVowels(const string& str);
```

The Contract

I give you a
string.

bool hasConsecutiveVowels(**const** string& str);

You tell me if it
has two or more
consecutive letters
that are vowels.

Trusting the Contract

```
bool isVowel(char ch);
```

```
bool hasConsecutiveVowels(const string& str) {
```

```
}
```

Trusting the Contract

```
bool isVowel(char ch);
```

```
bool hasConsecutiveVowels(const string& str) {  
    for (int i = 1; i < str.length(); i++) {  
  
    }  
  
}
```

Trusting the Contract

```
bool isVowel(char ch);
```

```
bool hasConsecutiveVowels(const string& str) {  
    for (int i = 1; i < str.length(); i++) {  
        if (str[i - 1] is a vowel && str[i] is a vowel) {  
            return true;  
        }  
    }  
}
```

Trusting the Contract

```
bool isVowel(char ch);
```

```
bool hasConsecutiveVowels(const string& str) {  
    for (int i = 1; i < str.length(); i++) {  
        if (str[i - 1] is a vowel && str[i] is a vowel) {  
            return true;  
        }  
    }  
    return false;  
}
```

Trusting the Contract

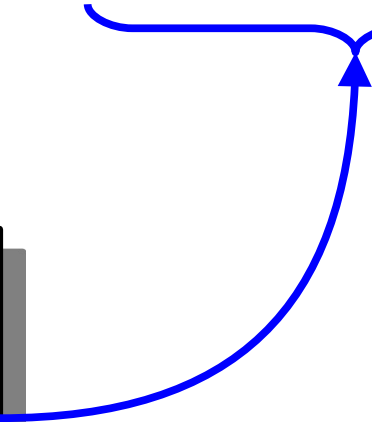
```
bool isVowel(char ch);
```

```
bool hasConsecutiveVowels(const string& str) {  
    for (int i = 1; i < str.length(); i++) {  
        if (isVowel(str[i - 1]) && isVowel(str[i])) {  
            return true;  
        }  
    }  
    return false;  
}
```

Trusting the Contract

```
bool isVowel(char ch);
```

```
bool hasConsecutiveVowels(const string& str) {  
    for (int i = 1; i < str.length(); i++) {  
        if (isVowel(str[i - 1]) && isVowel(str[i])) {  
            return true;  
        }  
    }  
    return false;  
}
```



It doesn't matter how
isVowel is implemented.
We just trust that it
works.

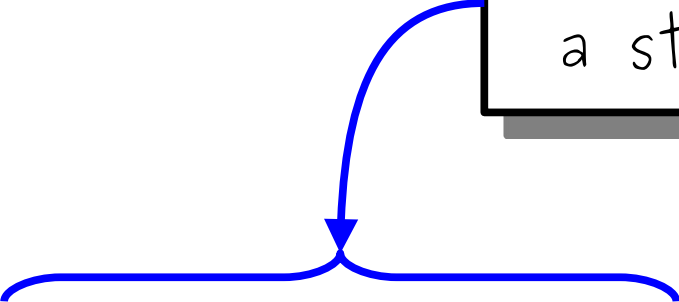
The Contract

The Contract

```
string reverseOf(const string& input);
```

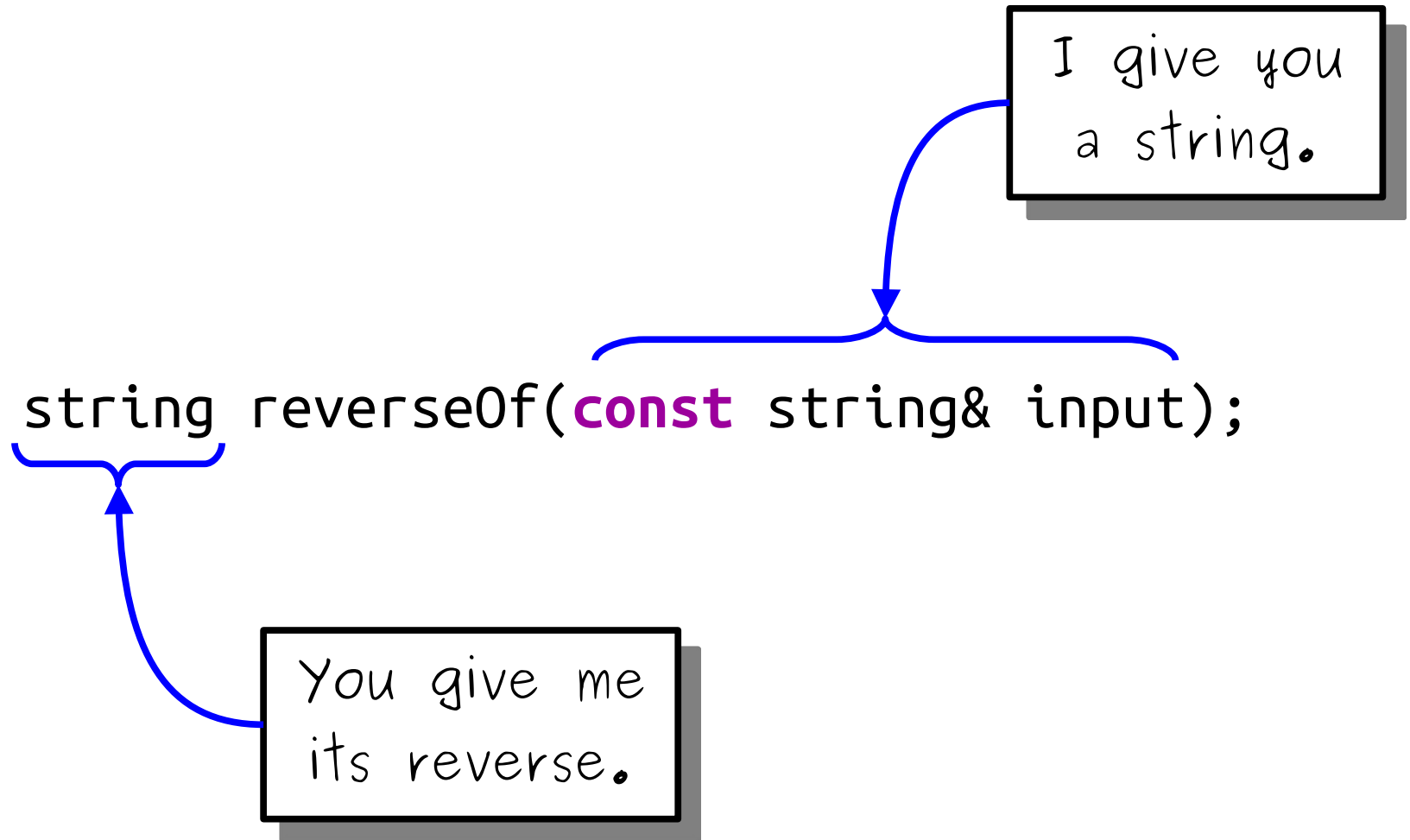
The Contract

I give you
a string.



```
string reverseOf(const string& input);
```

The Contract



Trusting the Contract

```
string reverseOf(const string& input);  
string reverseOf(const string& input) {  
  
}
```

Trusting the Contract

```
string reverseOf(const string& input);  
  
string reverseOf(const string& input) {  
    if (input == "") {  
  
    } else {  
  
    }  
}
```

Trusting the Contract

```
string reverseOf(const string& input);  
  
string reverseOf(const string& input) {  
    if (input == "") {  
        return "";  
    } else {  
  
    }  
}
```

Trusting the Contract

```
string reverseOf(const string& input);  
  
string reverseOf(const string& input) {  
    if (input == "") {  
        return "";  
    } else {  
        return the reverse of input.substr(1) + input[0];  
    }  
}
```

Trusting the Contract

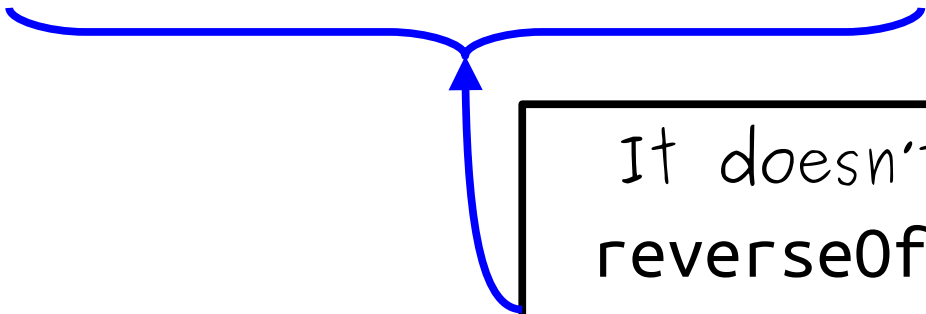
```
string reverseOf(const string& input);  
  
string reverseOf(const string& input) {  
    if (input == "") {  
        return "";  
    } else {  
        return reverseOf(input.substr(1)) + input[0];  
    }  
}
```

Trusting the Contract

```
string reverseOf(const string& input);  
  
string reverseOf(const string& input) {  
    if (input == "") {  
        return "";  
    } else {  
        return reverseOf(input.substr(1)) + input[0];  
    }  
}
```

Trusting the Contract

```
string reverseOf(const string& input);  
  
string reverseOf(const string& input) {  
    if (input == "") {  
        return "";  
    } else {  
        return reverseOf(input.substr(1)) + input[0];  
    }  
}
```



It doesn't matter how `reverseOf` reverses the string. It just matters that it does.

The Contract

The Contract

```
void drawTree(double x, double y,  
              double height,  
              double angle,  
              int order);
```

The Contract

```
void drawTree(double x, double y,  
              double height,  
              double angle,  
              int order);
```

The Contract

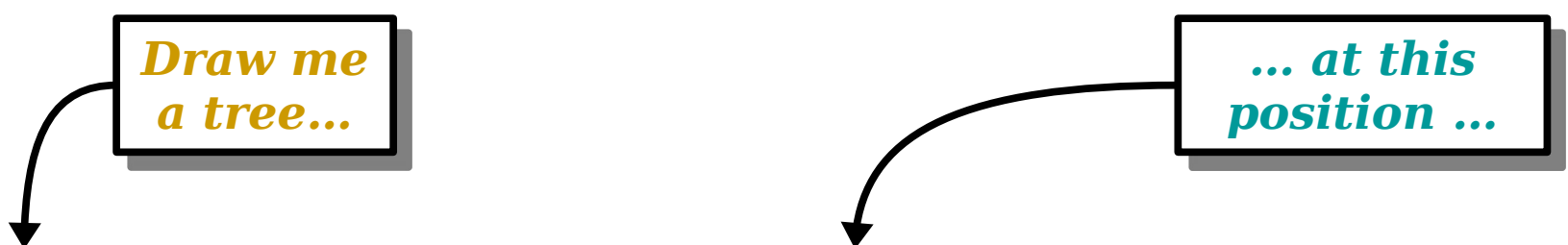
*Draw me
a tree...*



```
void drawTree(double x, double y,  
              double height,  
              double angle,  
              int order);
```

The Contract

*Draw me
a tree...*



*... at this
position ...*

```
void drawTree(double x, double y,  
              double height,  
              double angle,  
              int order);
```

The Contract

*Draw me
a tree...*

*... at this
position ...*

```
void drawTree(double x, double y,  
              double height,  
              double angle,  
              int order);
```

*... that's this
big ...*

The Contract

*Draw me
a tree...*

*... at this
position ...*

```
void drawTree(double x, double y,  
              double height,  
              double angle,  
              int order);
```

*... that's this
big ...*

*... facing
this way ...*

The Contract

*Draw me
a tree...*

*... at this
position ...*

```
void drawTree(double x, double y,  
              double height,  
              double angle,  
              int order);
```

*... that's this
big ...*

*... facing
this way ...*

*... with this
order.*

Trusting the Contract

```
void drawTree(double x, double y,  
              double height, double angle,  
              int order);
```

```
void drawTree(double x, double y,  
              double height, double angle,  
              int order) {
```

```
}
```

Trusting the Contract

```
void drawTree(double x, double y,  
              double height, double angle,  
              int order);
```

```
void drawTree(double x, double y,  
              double height, double angle,  
              int order) {  
    if (order == 0) return;
```

```
}
```

Trusting the Contract

```
void drawTree(double x, double y,  
              double height, double angle,  
              int order);
```

```
void drawTree(double x, double y,  
              double height, double angle,  
              int order) {  
    if (order == 0) return;  
  
    GPoint endpoint = drawPolarLine(/* ... */);  
  
}
```

Trusting the Contract

```
void drawTree(double x, double y,  
              double height, double angle,  
              int order);
```

```
void drawTree(double x, double y,  
              double height, double angle,  
              int order) {  
    if (order == 0) return;
```

```
    GPoint endpoint = drawPolarLine(/* ... */);
```

```
    draw a tree angling to the left  
    draw a tree angling to the right
```

```
}
```

Trusting the Contract

```
void drawTree(double x, double y,  
              double height, double angle,  
              int order);
```

```
void drawTree(double x, double y,  
              double height, double angle,  
              int order) {  
    if (order == 0) return;
```

```
    GPoint endpoint = drawPolarLine(/* ... */);
```

```
    drawTree(/* ... */);
```

```
    drawTree(/* ... */);
```

```
}
```

Trusting the Contract

```
void drawTree(double x, double y,  
             double height, double angle,  
             int order);
```

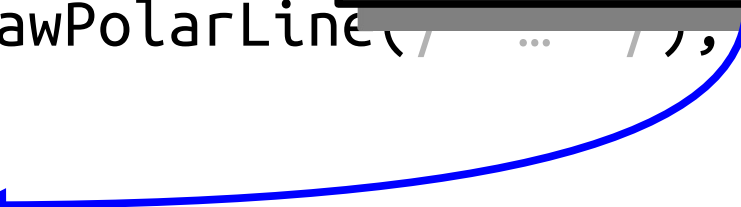
```
void drawTree(double x, double y,  
             double height, double angle,  
             int order) {  
    if (order == 0) return;
```

```
    GPoint endpoint = drawPolarLine(x, y, height, angle, order);
```

```
    drawTree(endpoint.x, endpoint.y, height, angle, order + 1);  
    drawTree(endpoint.x, endpoint.y, height, angle, order + 1);
```

```
}
```

It doesn't matter how
drawTree draws a
tree. It just matters
that it does.



The Recursive Leap of Faith

- When writing a recursive function, it helps to take a ***recursive leap of faith***.
- Before writing the function, answer these questions:
 - What does the function take in?
 - What does it return?
- Then, as you're writing the function, trust that your recursive calls to the function just "work" without asking how.
- This can take some adjustment to get used to, but is a necessary skill for writing more complex recursive functions.

Time-Out for Announcements!

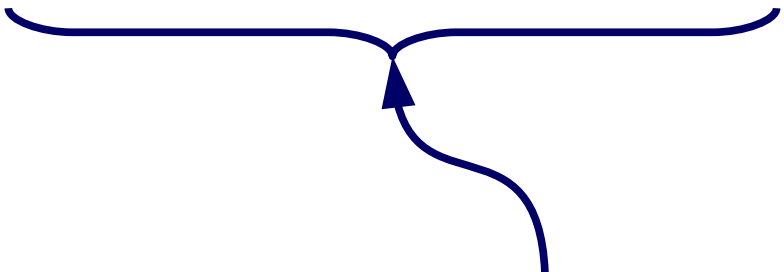
Recursive Drawing Contest

- We are holding a (purely optional, just for fun) Recursive Drawing contest!
- Visit <http://recursivedrawing.com/>, draw whatever you'd like, and post it to the EdStem thread for the contest.
- We'll award recursion-themed prizes to a small number of entries.
- Deadline to submit is Monday at 1:00PM Pacific.

Assignment 2

- Assignment 2 is due this Friday at 1:00PM.
 - If you're following our timetable, you'll have finished Rosetta Stone at this point and be midway through Rising Tides.
- Have questions?

Recursive Enumeration



e·nu·mer·a·tion

noun

The act of mentioning a number of things one by one.

(Source: Oxford Languages, via Google)

Listing Subsets

- A set S is a **subset** of a set T when every element of S is an element of T .
- There are two subsets of $\{2\}$:
 $\{ \}$ $\{2\}$
- There are four subsets of $\{2, 3\}$:
 $\{ \}$ $\{2\}$ $\{3\}$ $\{2, 3\}$
- How many subsets are there of $\{2, 3, 4\}$?

Listing Subsets

- A set S is a **subset** of a set T when every element of S is an element of T .

- There are two subsets of $\{2\}$:

$\{ \}$ $\{2\}$

- There are four subsets of $\{2, 3\}$:

$\{ \}$ $\{2\}$ $\{3\}$ $\{2, 3\}$

- How many subsets are there of $\{2, 3, 4\}$?

$\{ \}$
 $\{2\}$ $\{3\}$ $\{4\}$
 $\{2, 3\}$ $\{2, 4\}$ $\{3, 4\}$
 $\{2, 3, 4\}$

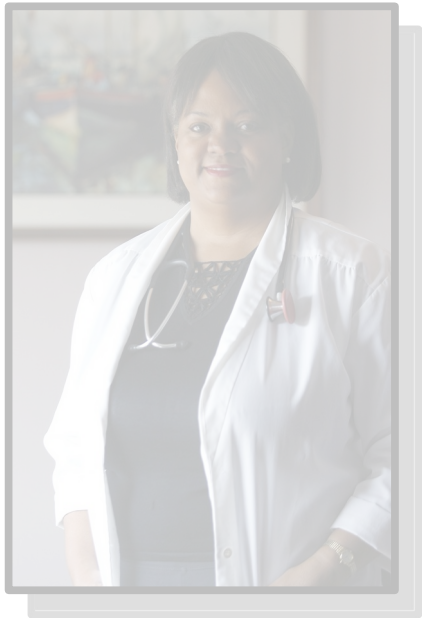
- The only subset of $\{ \}$ is $\{ \}$.

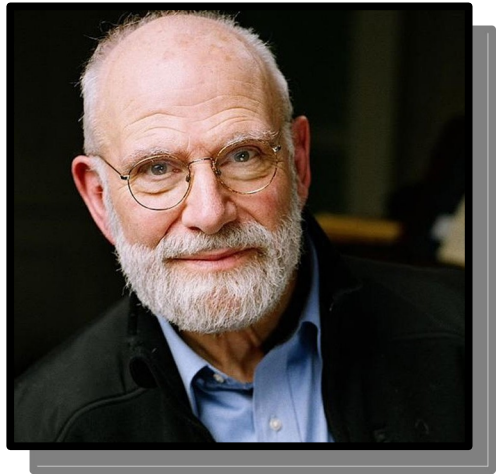


You need to send an emergency team of doctors to an area.

You know which doctors you have available to send.

List all the possible teams you can make from your list of all the doctors.





1

2

3

4

{ 1, 2, 3, 4 }

$\{ 1, 2, 3, 4 \}$

$\{$
 $\{$
 $\{ \quad 4 \}$
 $\{ \quad \quad 3 \}$
 $\{ \quad \quad 3, 4 \}$
 $\{ \quad 2 \}$
 $\{ \quad 2, \quad 4 \}$
 $\{ \quad 2, 3 \}$
 $\{ \quad 2, 3, 4 \}$
 $\}$

$\{$
 $\{ \quad 1 \}$
 $\{ \quad 1, \quad 4 \}$
 $\{ \quad 1, \quad 3 \}$
 $\{ \quad 1, \quad 3, 4 \}$
 $\{ \quad 1, 2 \}$
 $\{ \quad 1, 2, \quad 4 \}$
 $\{ \quad 1, 2, 3 \}$
 $\{ \quad 1, 2, 3, 4 \}$
 $\}$

$$\{ 1, 2, 3, 4 \}$$
$$\left\{ \begin{array}{l} \{ \\ \{ \\ \{ \\ \{ \\ \{ \\ \{ \\ \{ \\ \{ \end{array} \right\}$$

These are all the
subsets of
 $\{ 2, 3, 4 \}$.

{ 1 }
 { 1 , 4 }
 { 1 , 3 }
 { 3 , 4 }
 { 1 , 2 , 4 }
 { 1 , 2 , 3 }
 { 1 , 2 , 3 , 4 }

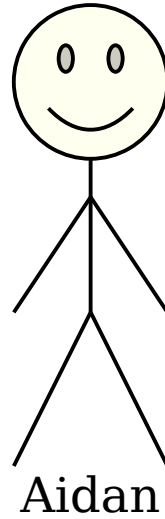
$\{ 1, 2, 3, 4 \}$

These are all the
subsets of
 $\{ 2, 3, 4 \}$ with 1
inserted into them.

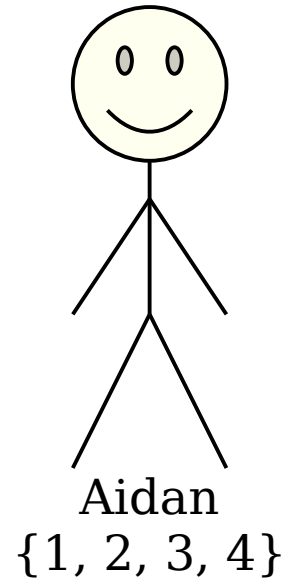
$\{ 1 \}$
 $\{ 1, 4 \}$
 $\{ 1, 3 \}$
 $\{ 1, 3, 4 \}$
 $\{ 1, 2 \}$
 $\{ 1, 2, 4 \}$
 $\{ 1, 2, 3 \}$
 $\{ 1, 2, 3, 4 \}$

Aidans List Subsets

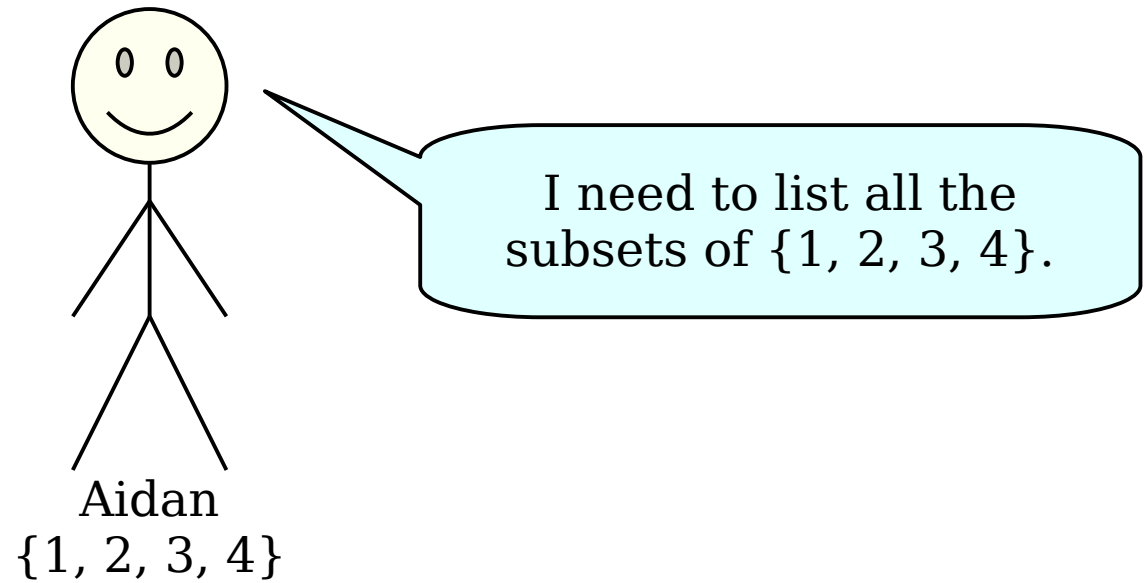
Aidans List Subsets



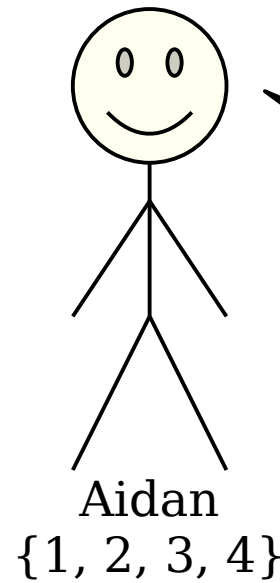
Aidans List Subsets



Aidans List Subsets

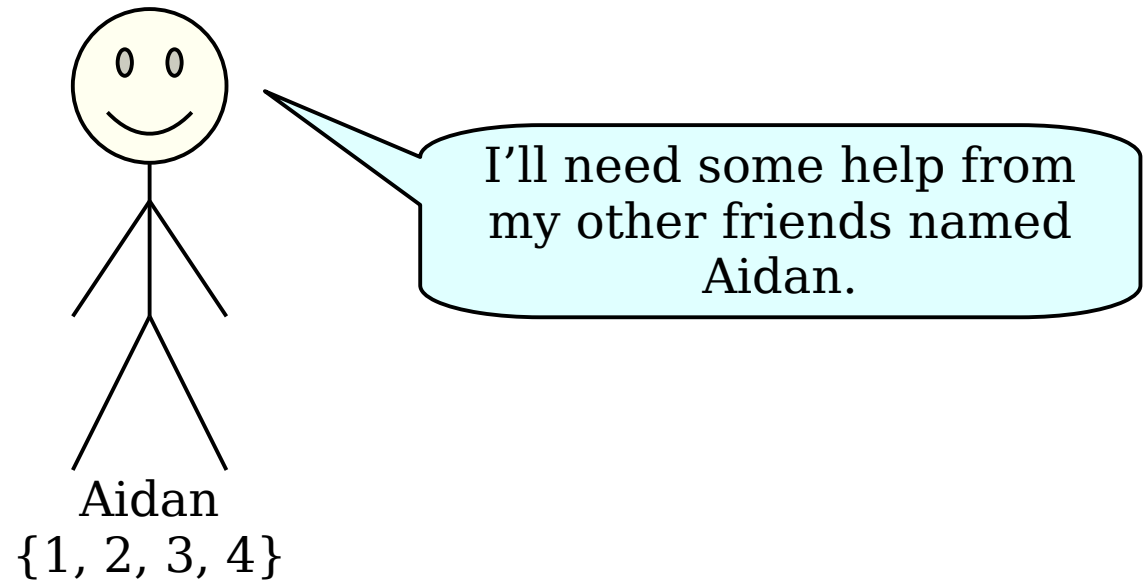


Aidans List Subsets

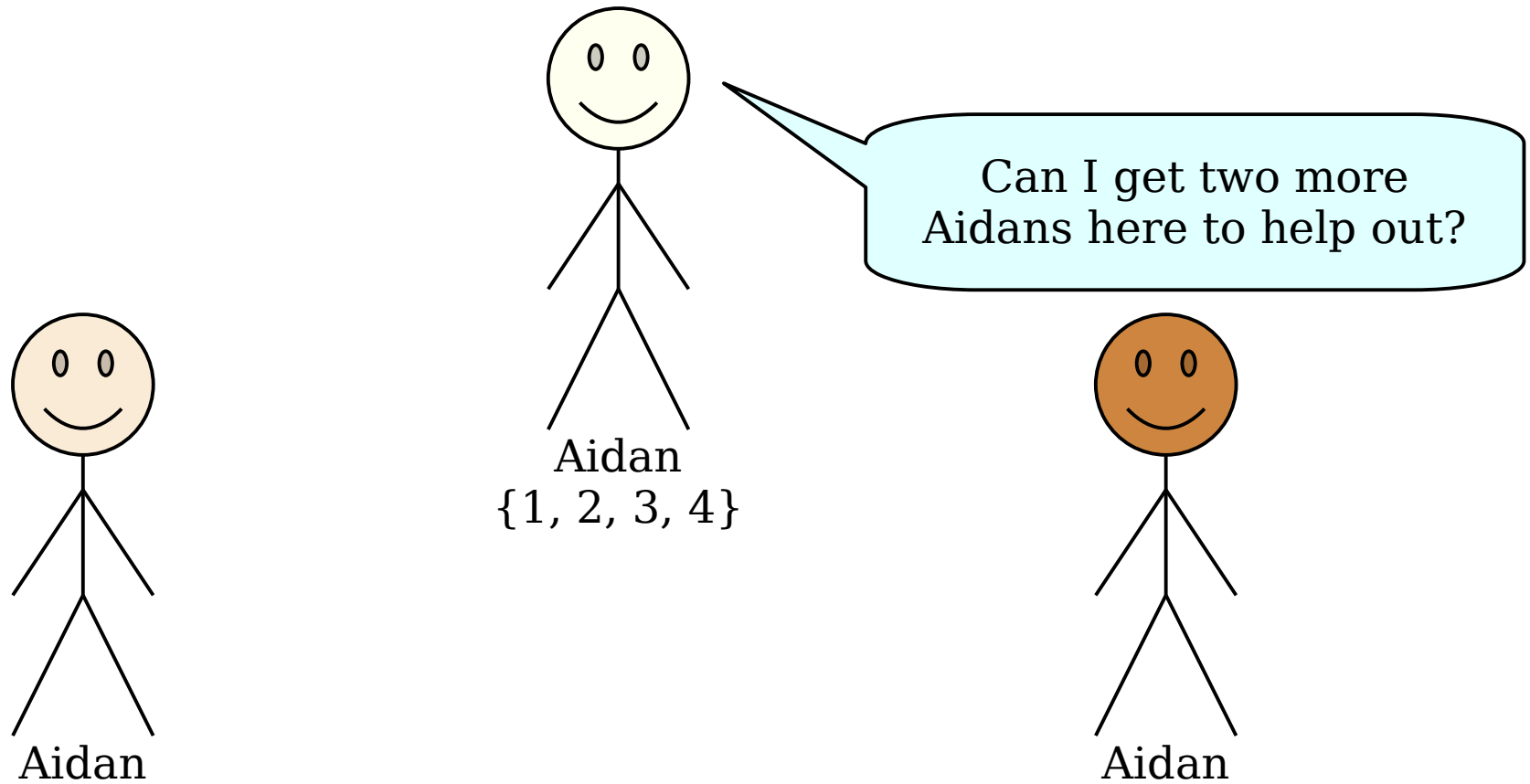


Each of those subsets
either includes 1 or
doesn't include 1.

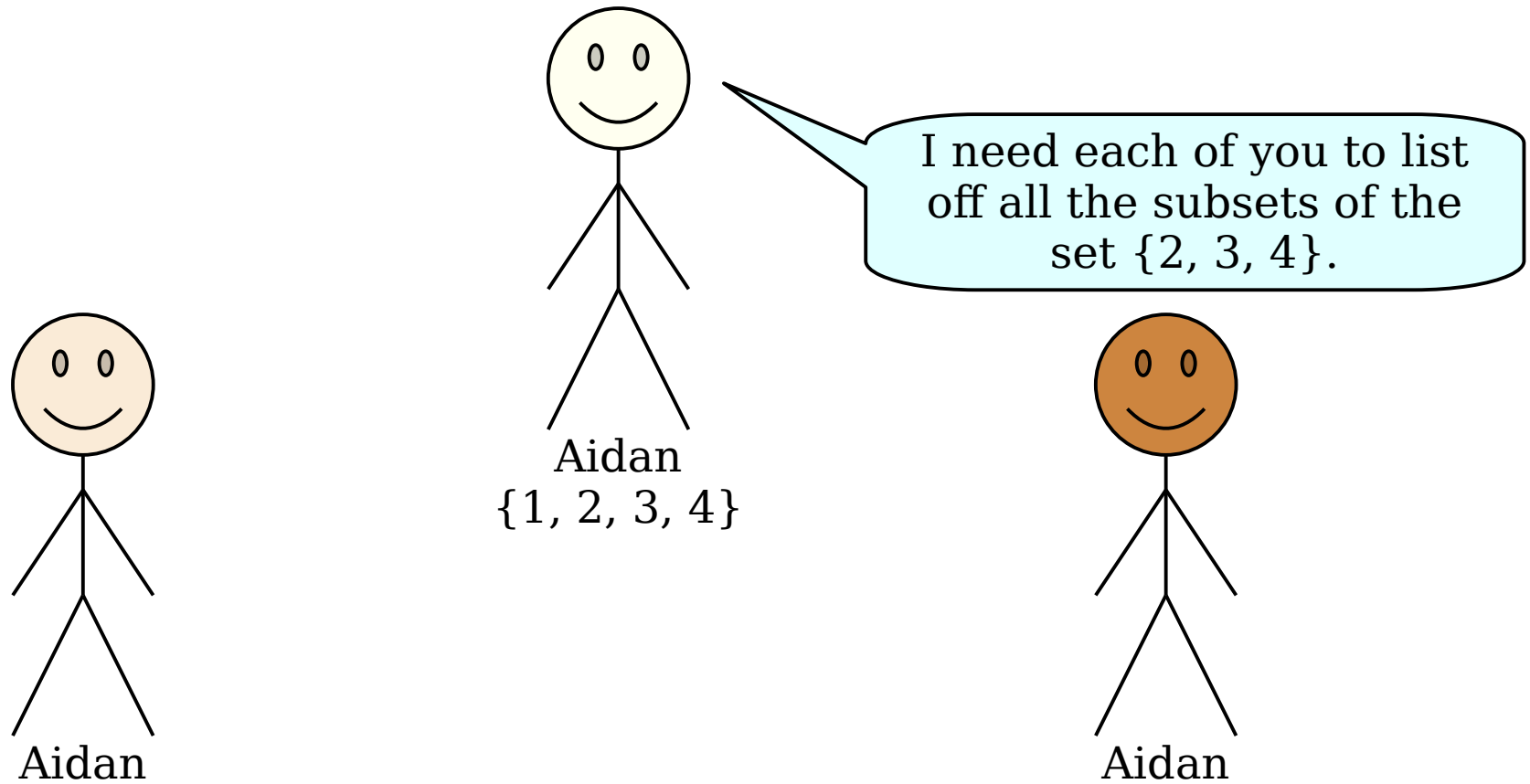
Aidans List Subsets



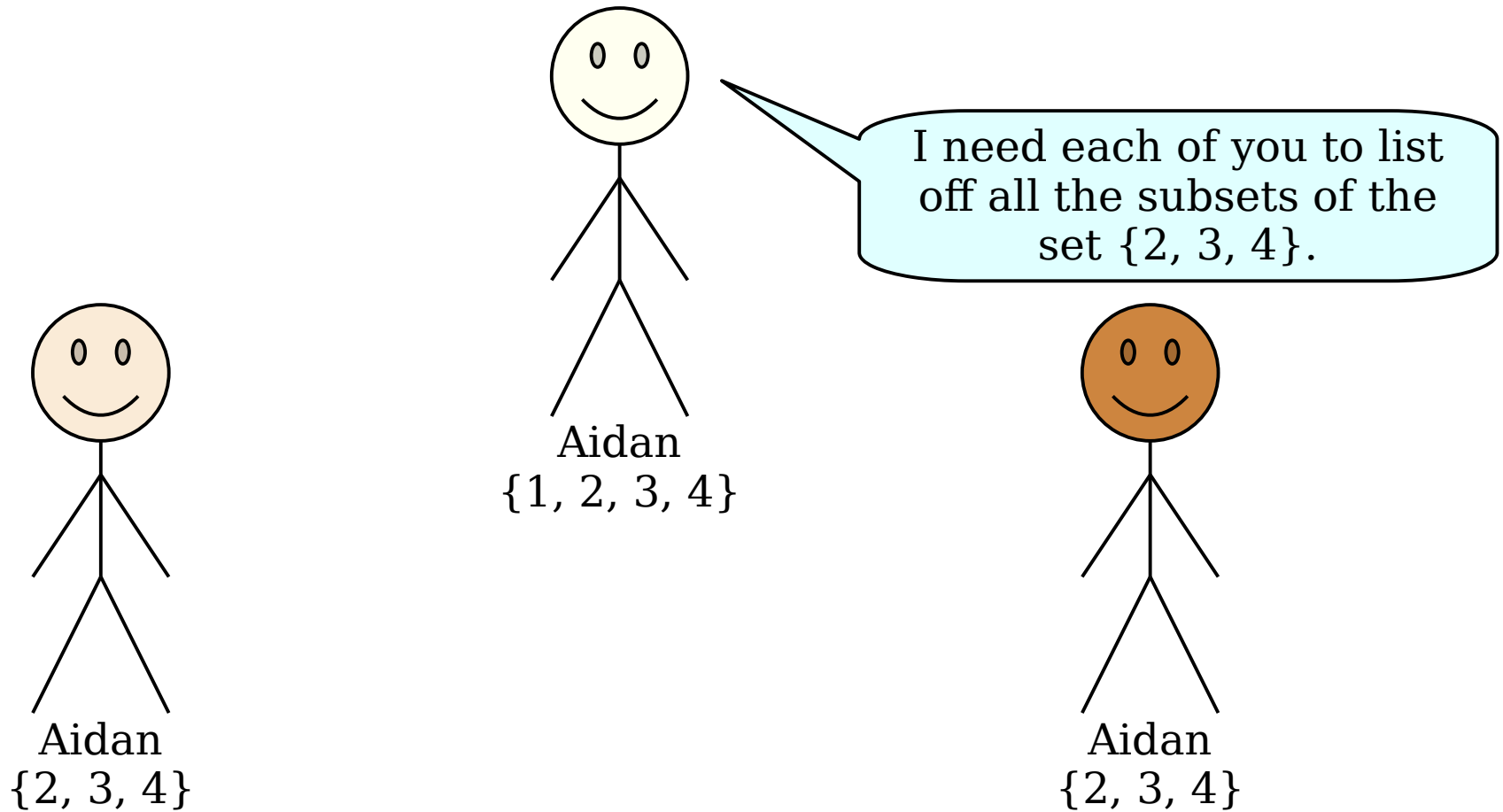
Aidans List Subsets



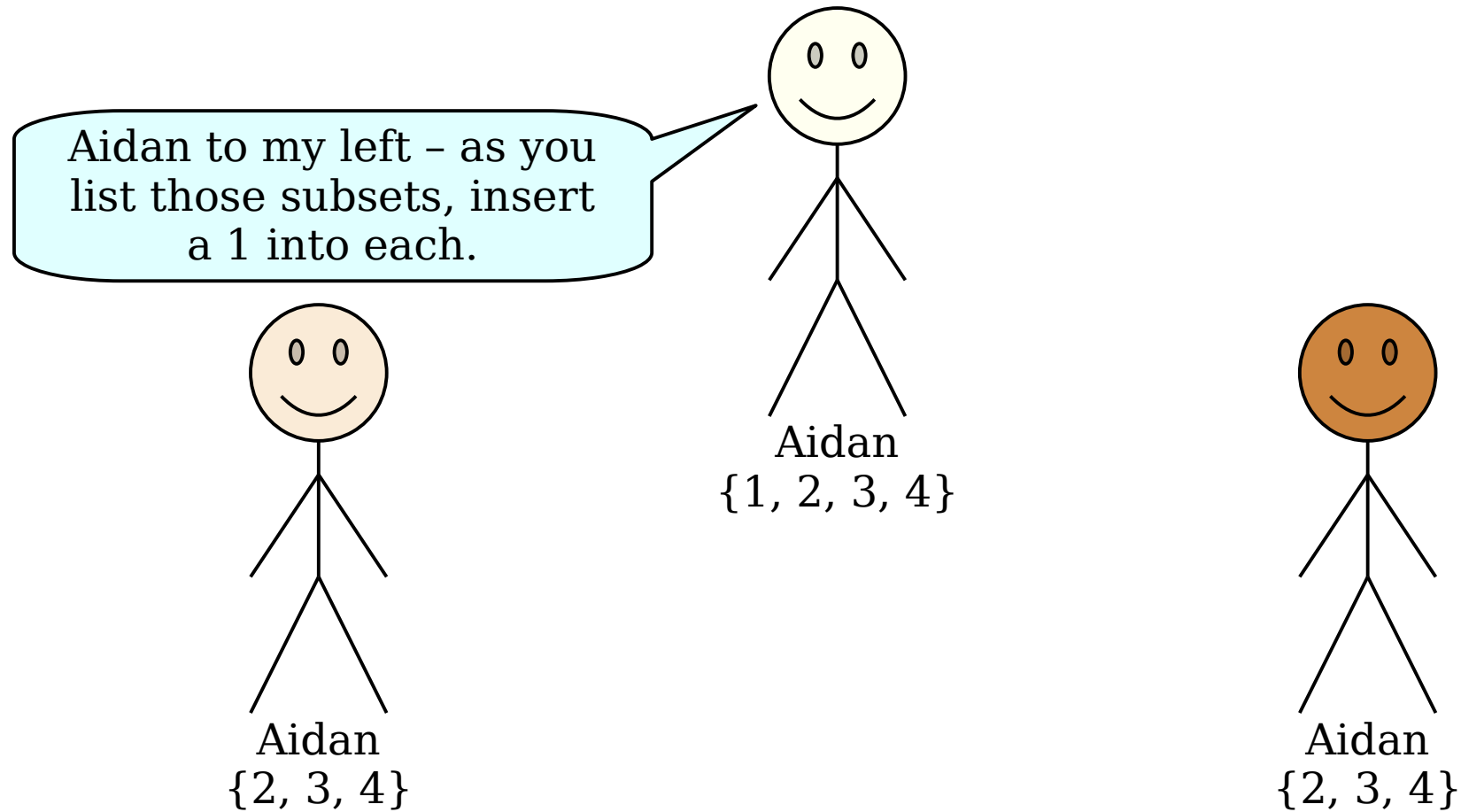
Aidans List Subsets



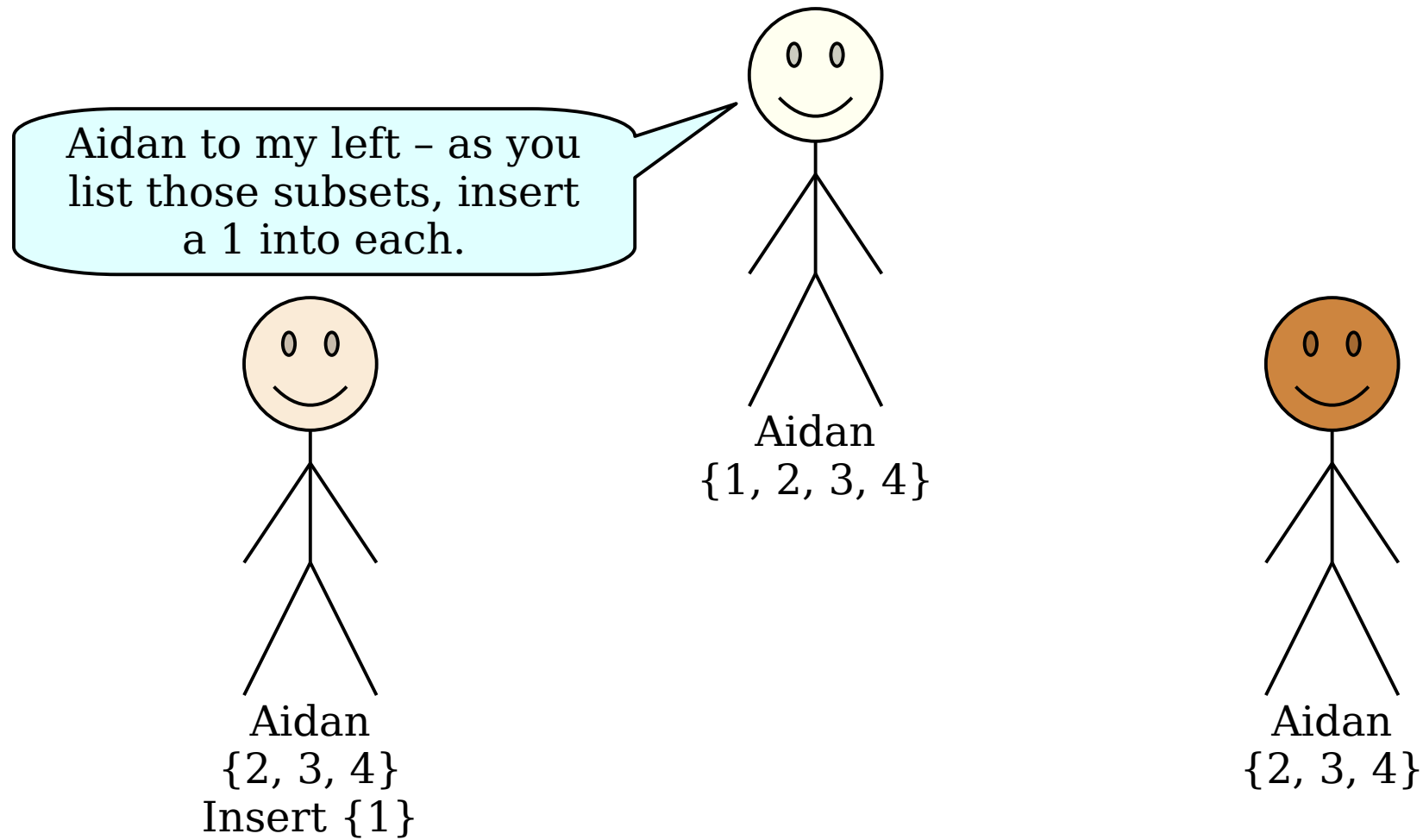
Aidans List Subsets



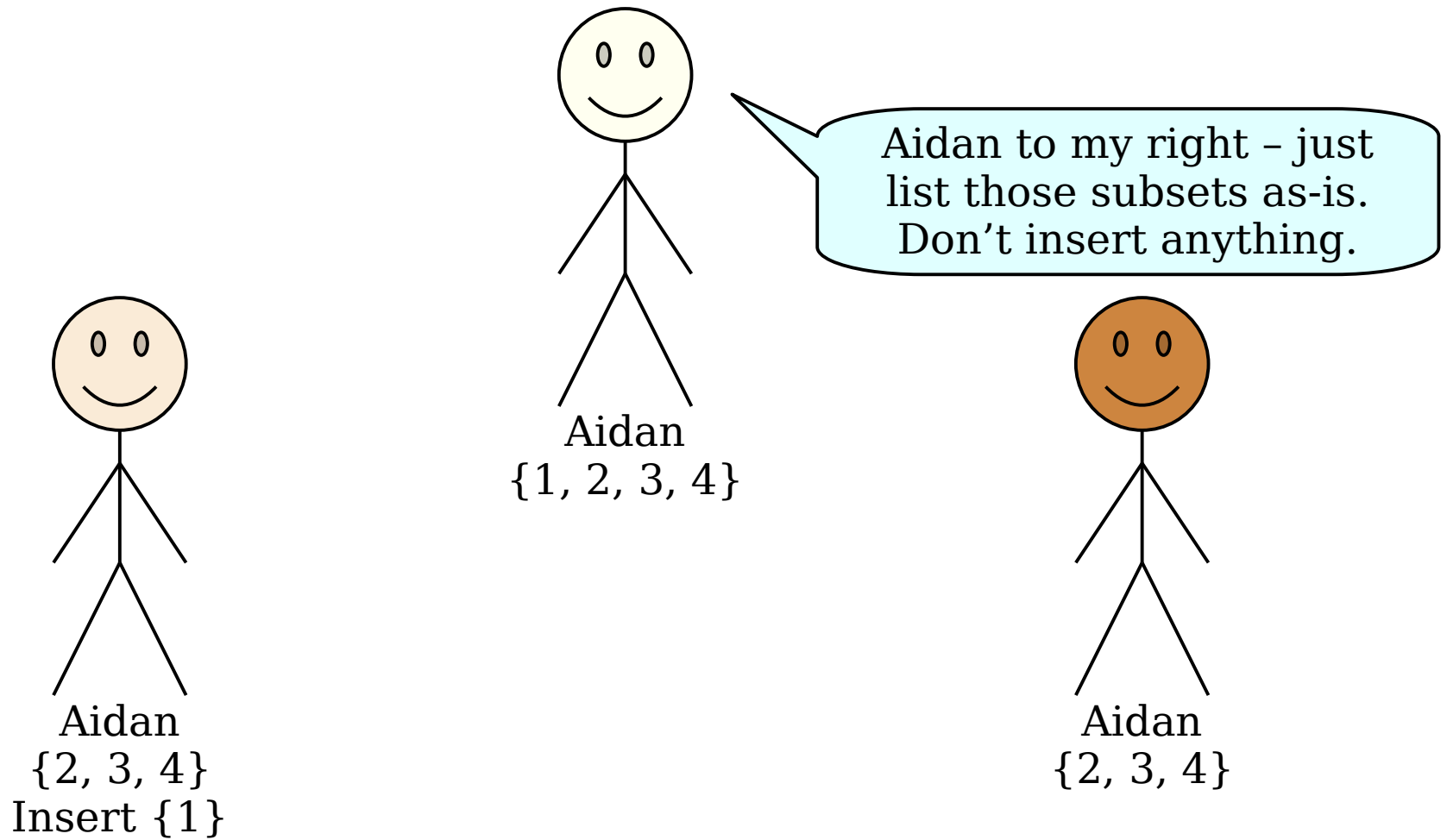
Aidans List Subsets



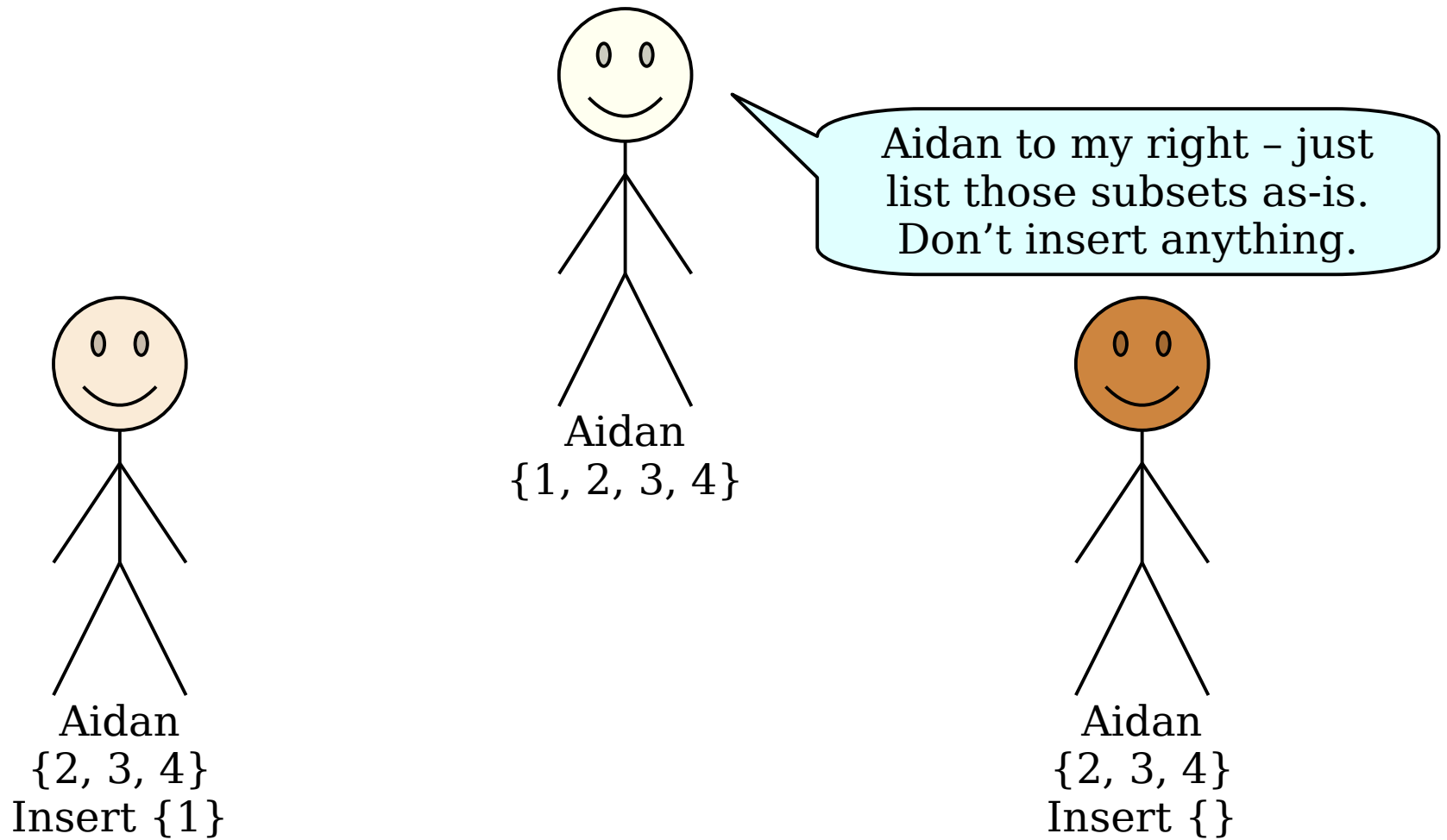
Aidans List Subsets



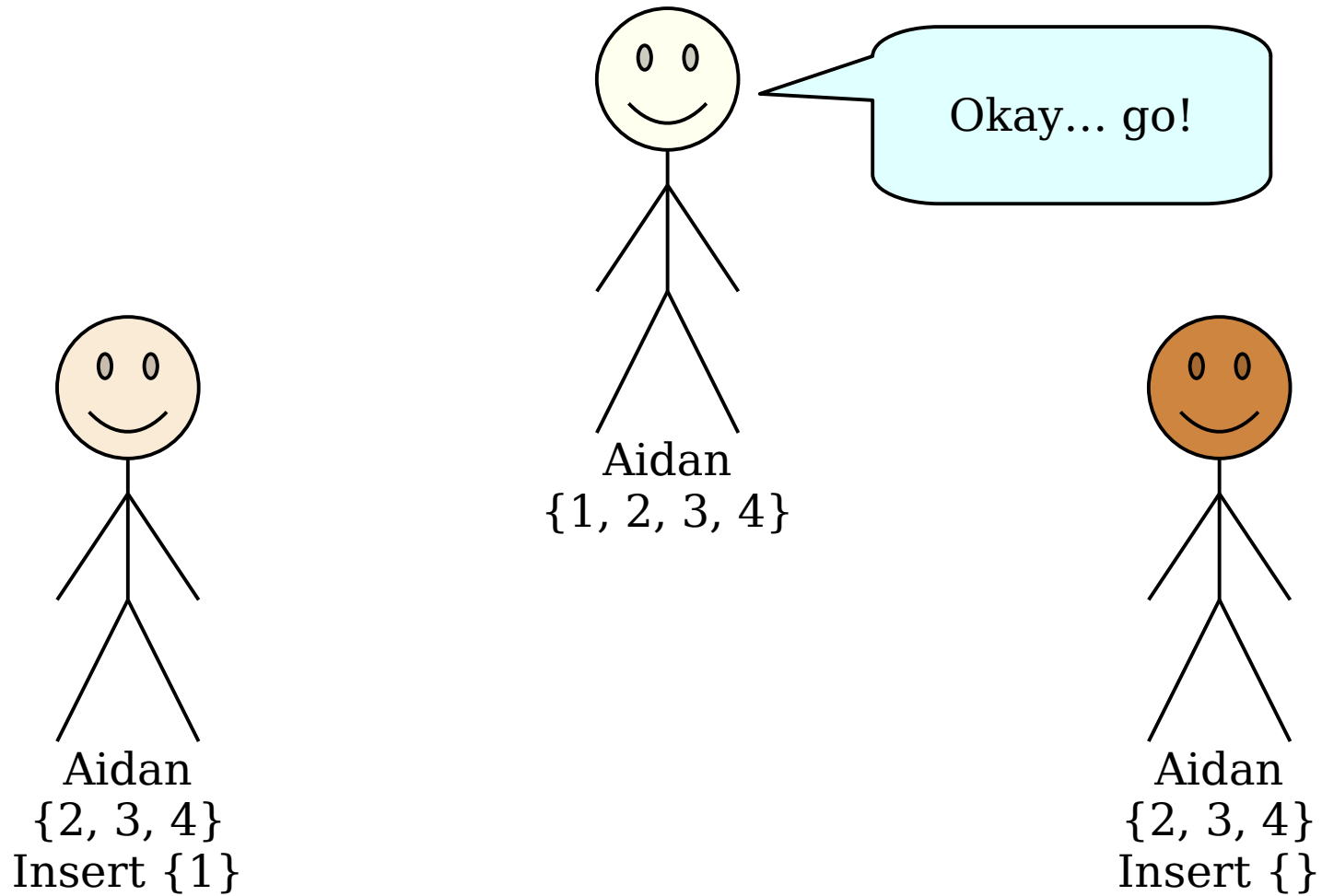
Aidans List Subsets



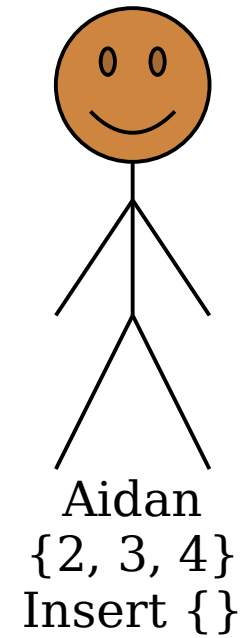
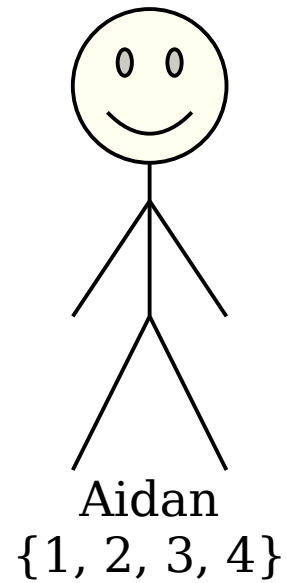
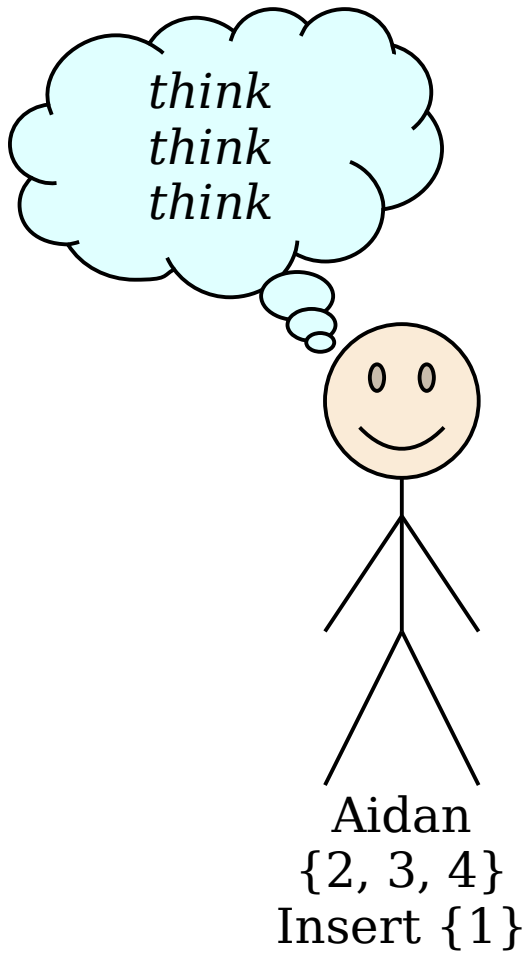
Aidans List Subsets



Aidans List Subsets

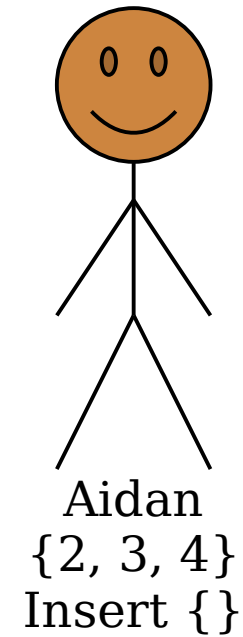
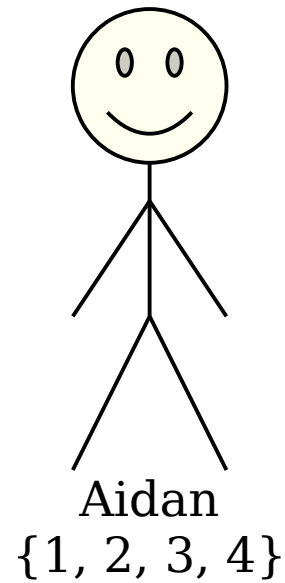
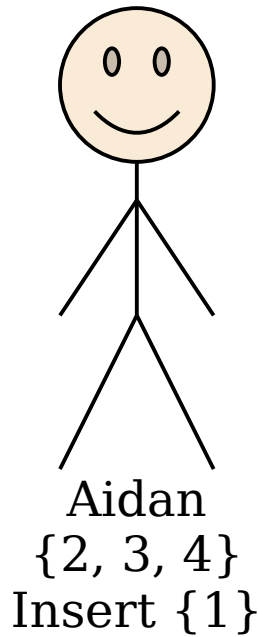


Aidans List Subsets



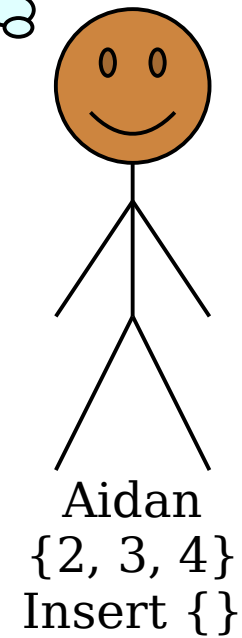
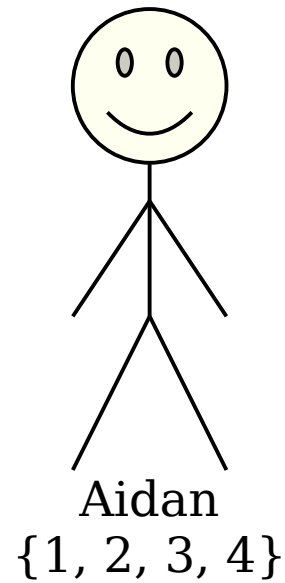
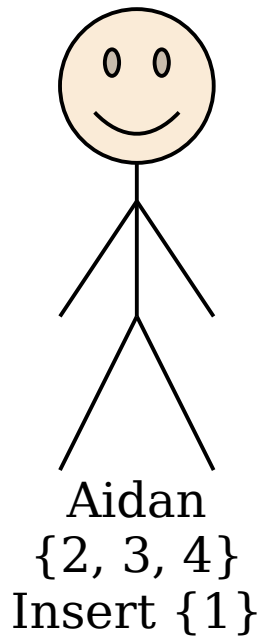
Aidans List Subsets

Here's what
you asked for!



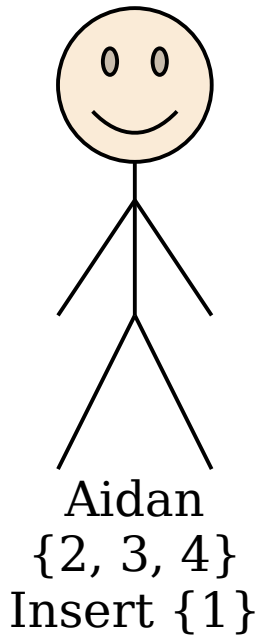
$\{1\}$	$\{1, 2\}$
$\{1, 4\}$	$\{1, 2, 4\}$
$\{1, 3\}$	$\{1, 2, 3\}$
$\{1, 3, 4\}$	$\{1, 2, 3, 4\}$

Aidans List Subsets

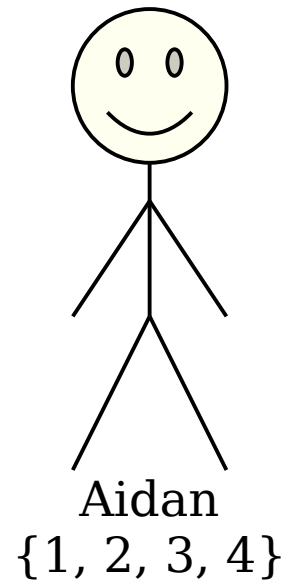


$\{1\}$	$\{1, 2\}$
$\{1, 4\}$	$\{1, 2, 4\}$
$\{1, 3\}$	$\{1, 2, 3\}$
$\{1, 3, 4\}$	$\{1, 2, 3, 4\}$

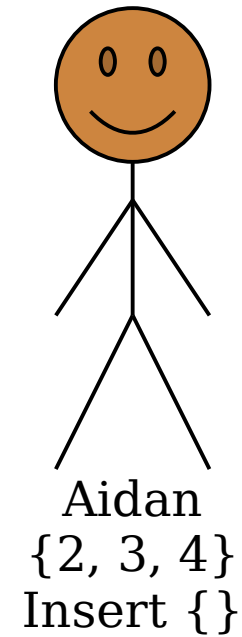
Aidans List Subsets



$\{1\}$	$\{1, 2\}$
$\{1, 4\}$	$\{1, 2, 4\}$
$\{1, 3\}$	$\{1, 2, 3\}$
$\{1, 3, 4\}$	$\{1, 2, 3, 4\}$



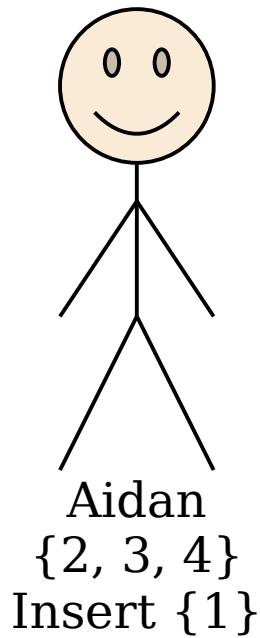
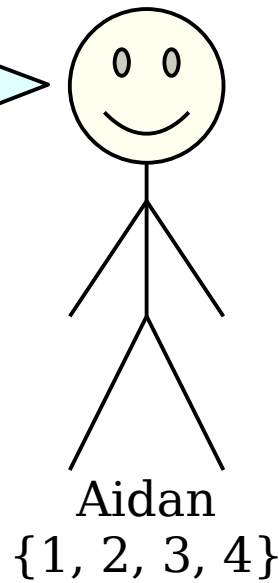
Here's what
you asked for!



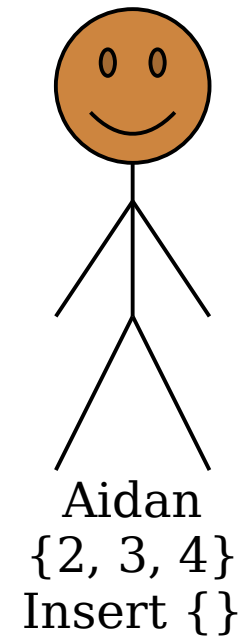
$\{\}$	$\{2\}$
$\{4\}$	$\{2, 4\}$
$\{3\}$	$\{2, 3\}$
$\{3, 4\}$	$\{2, 3, 4\}$

Aidans List Subsets

Thanks! You
guys are
great.

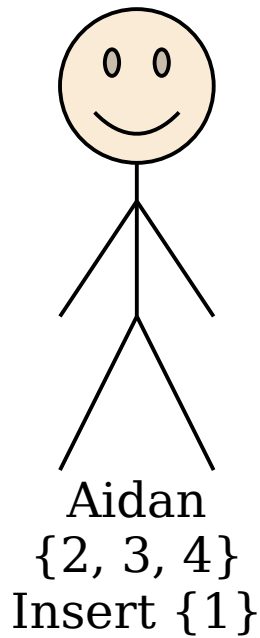
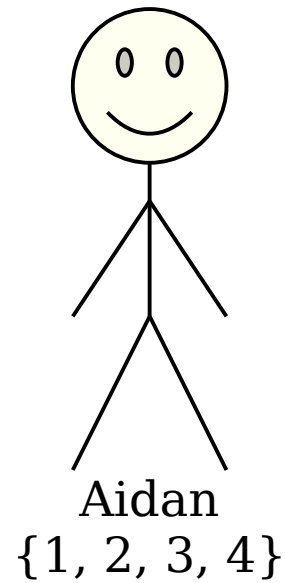


$\{1\}$	$\{1, 2\}$
$\{1, 4\}$	$\{1, 2, 4\}$
$\{1, 3\}$	$\{1, 2, 3\}$
$\{1, 3, 4\}$	$\{1, 2, 3, 4\}$

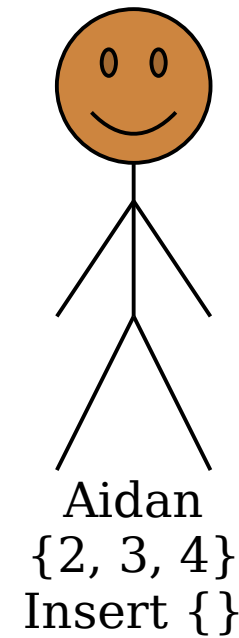


$\{\}$	$\{2\}$
$\{4\}$	$\{2, 4\}$
$\{3\}$	$\{2, 3\}$
$\{3, 4\}$	$\{2, 3, 4\}$

Aidans List Subsets



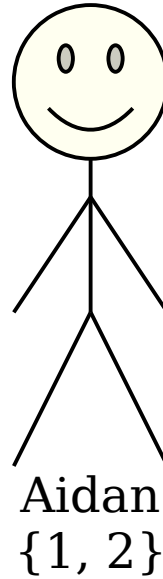
$\{1\}$	$\{1, 2\}$
$\{1, 4\}$	$\{1, 2, 4\}$
$\{1, 3\}$	$\{1, 2, 3\}$
$\{1, 3, 4\}$	$\{1, 2, 3, 4\}$



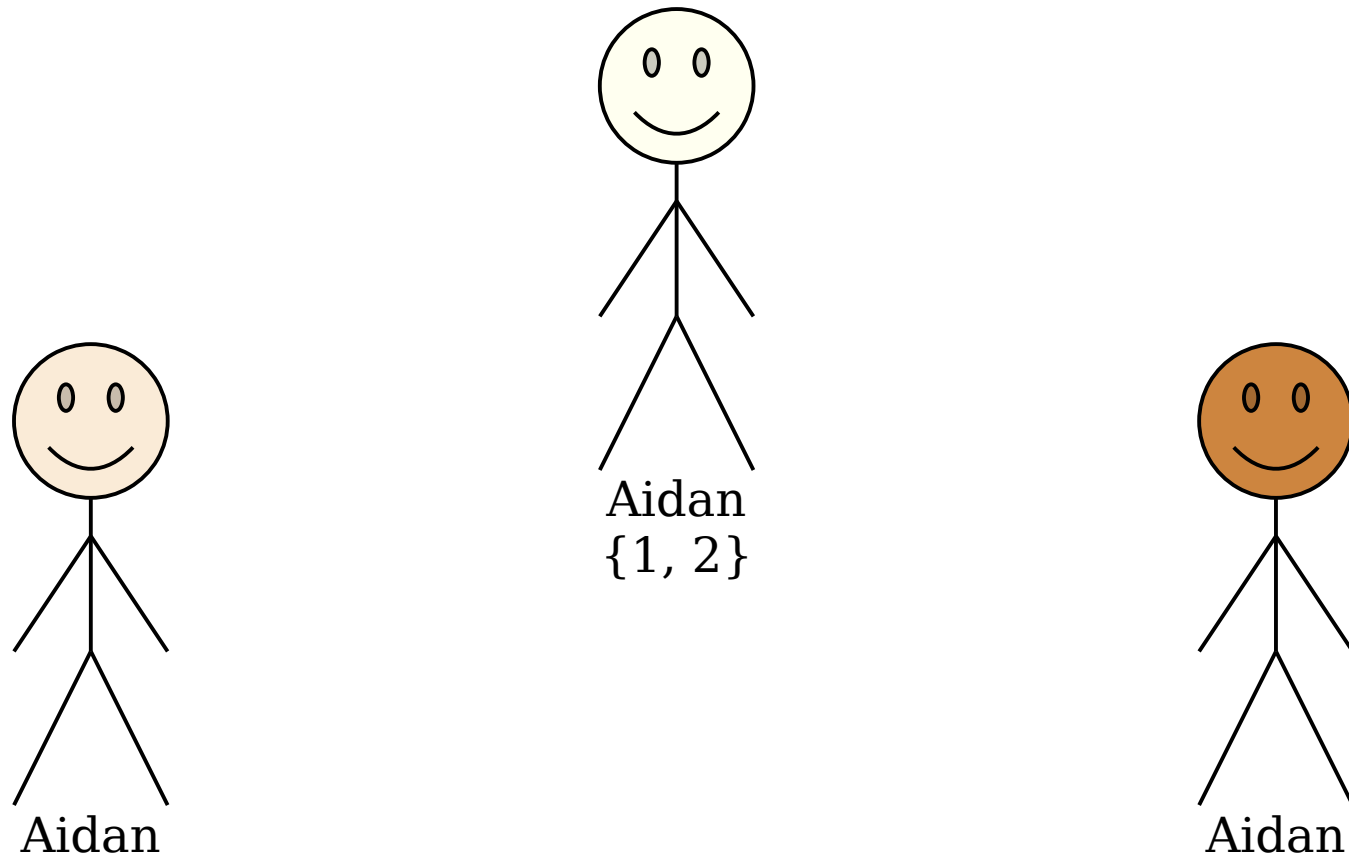
$\{\}$	$\{2\}$
$\{4\}$	$\{2, 4\}$
$\{3\}$	$\{2, 3\}$
$\{3, 4\}$	$\{2, 3, 4\}$

Thinking Recursively

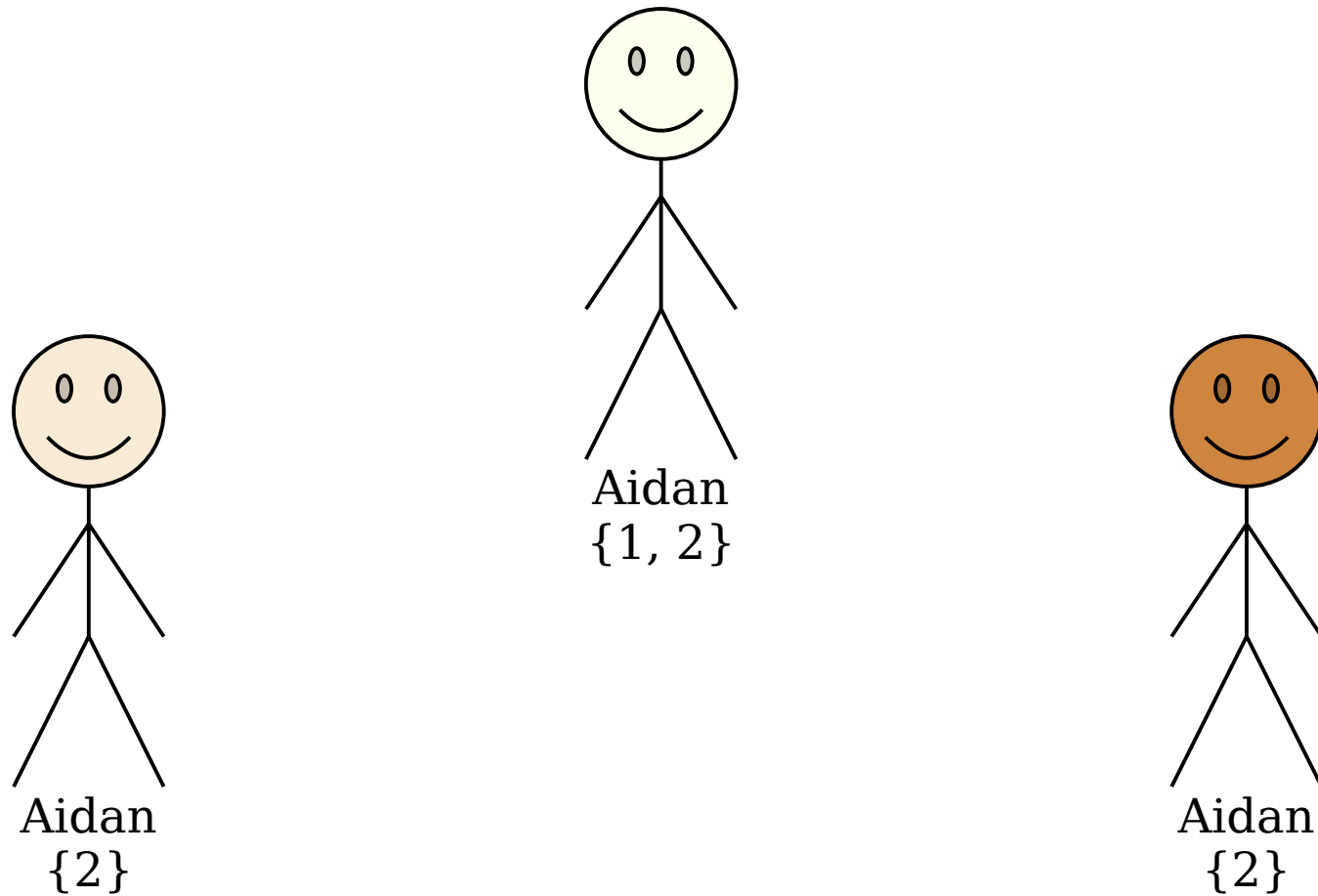
Aidans List Subsets



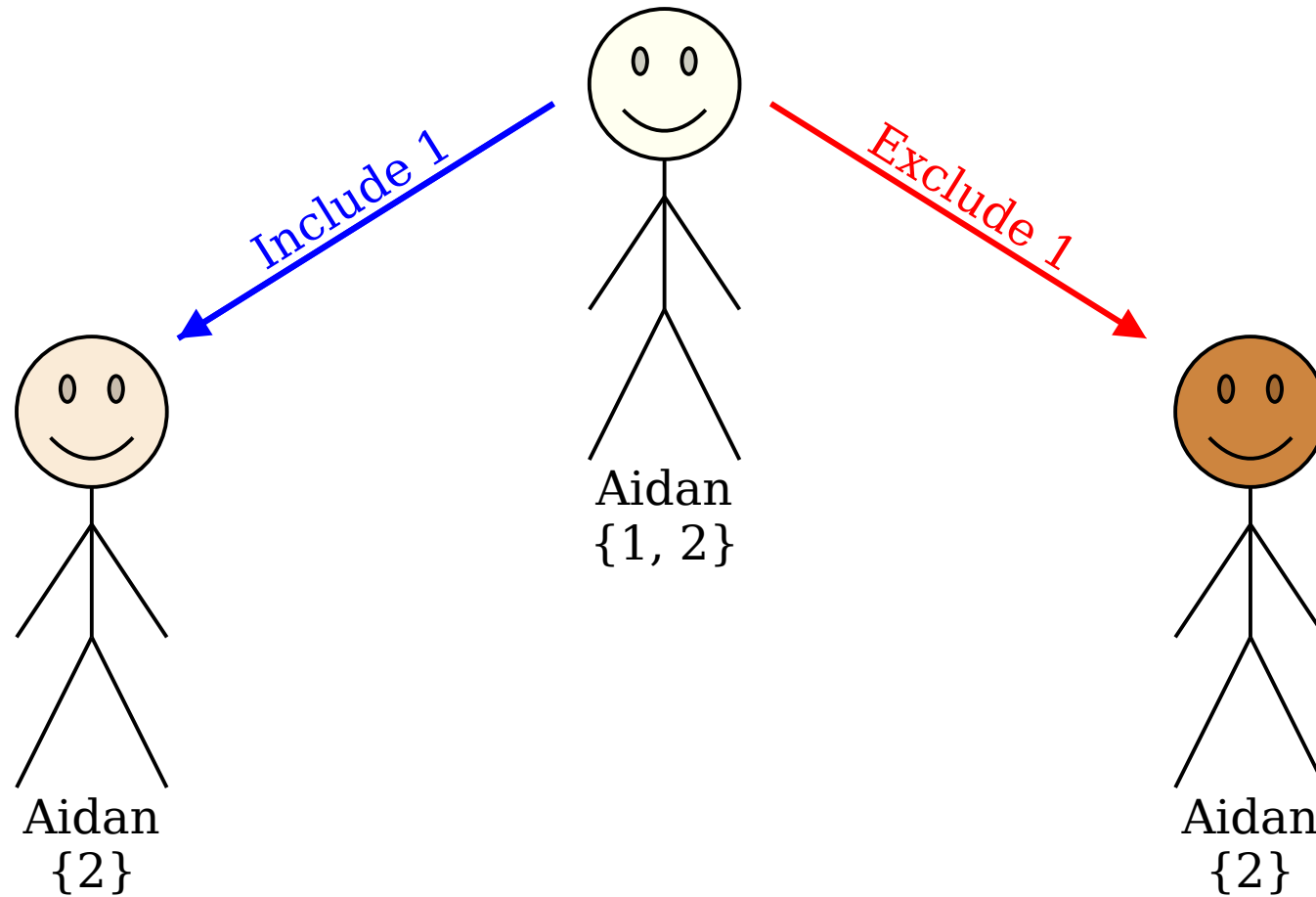
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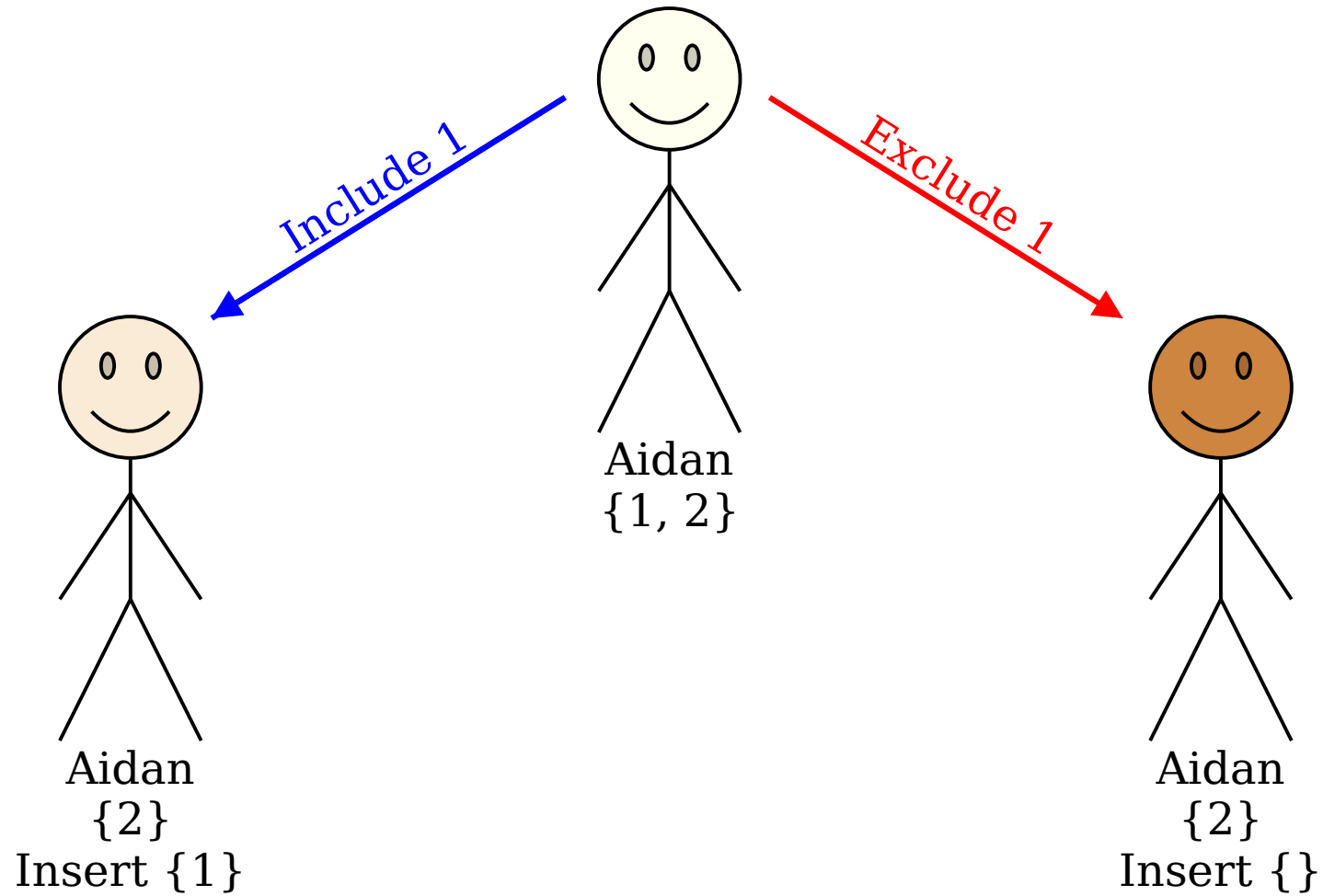
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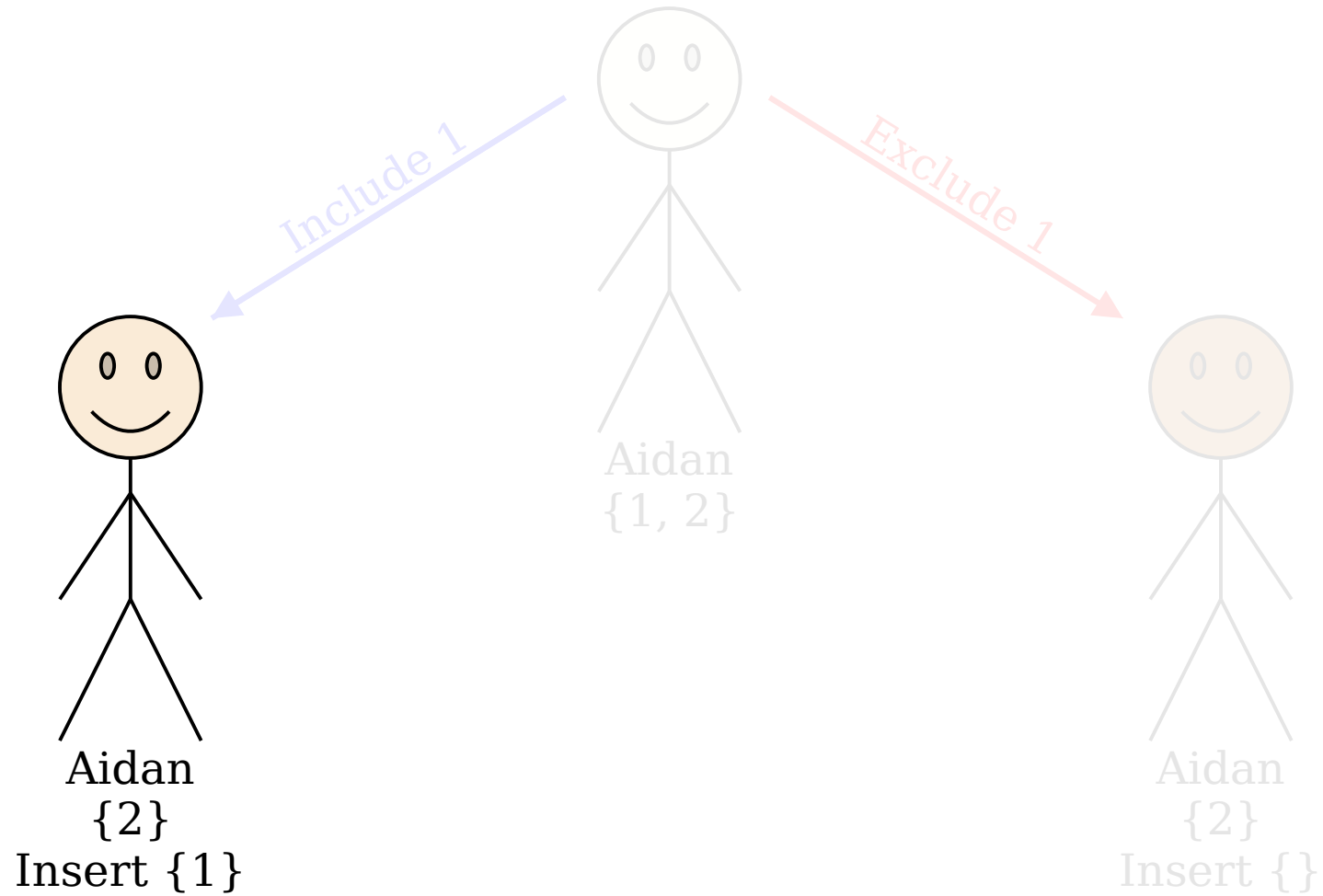
Aidans List Subsets



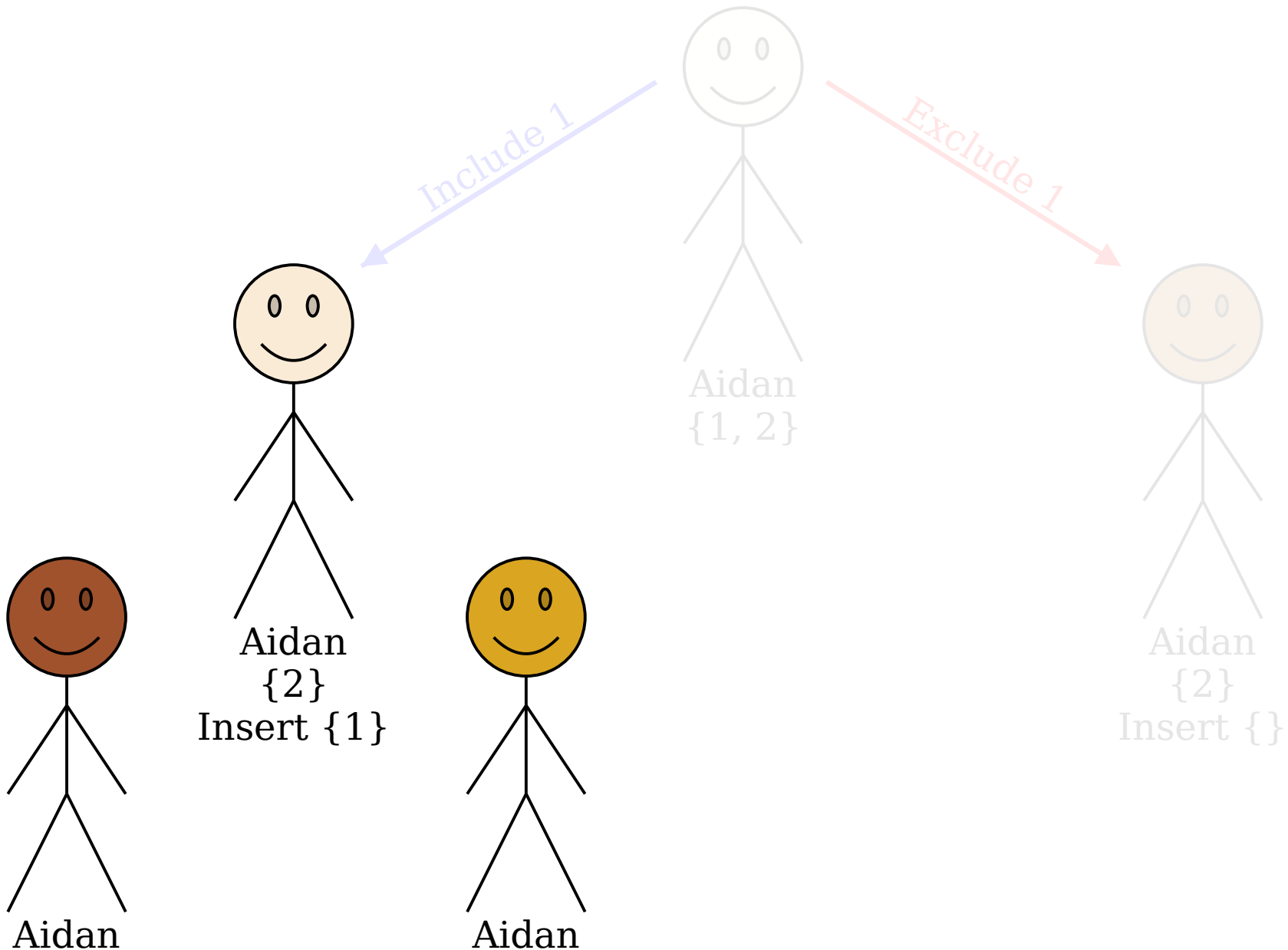
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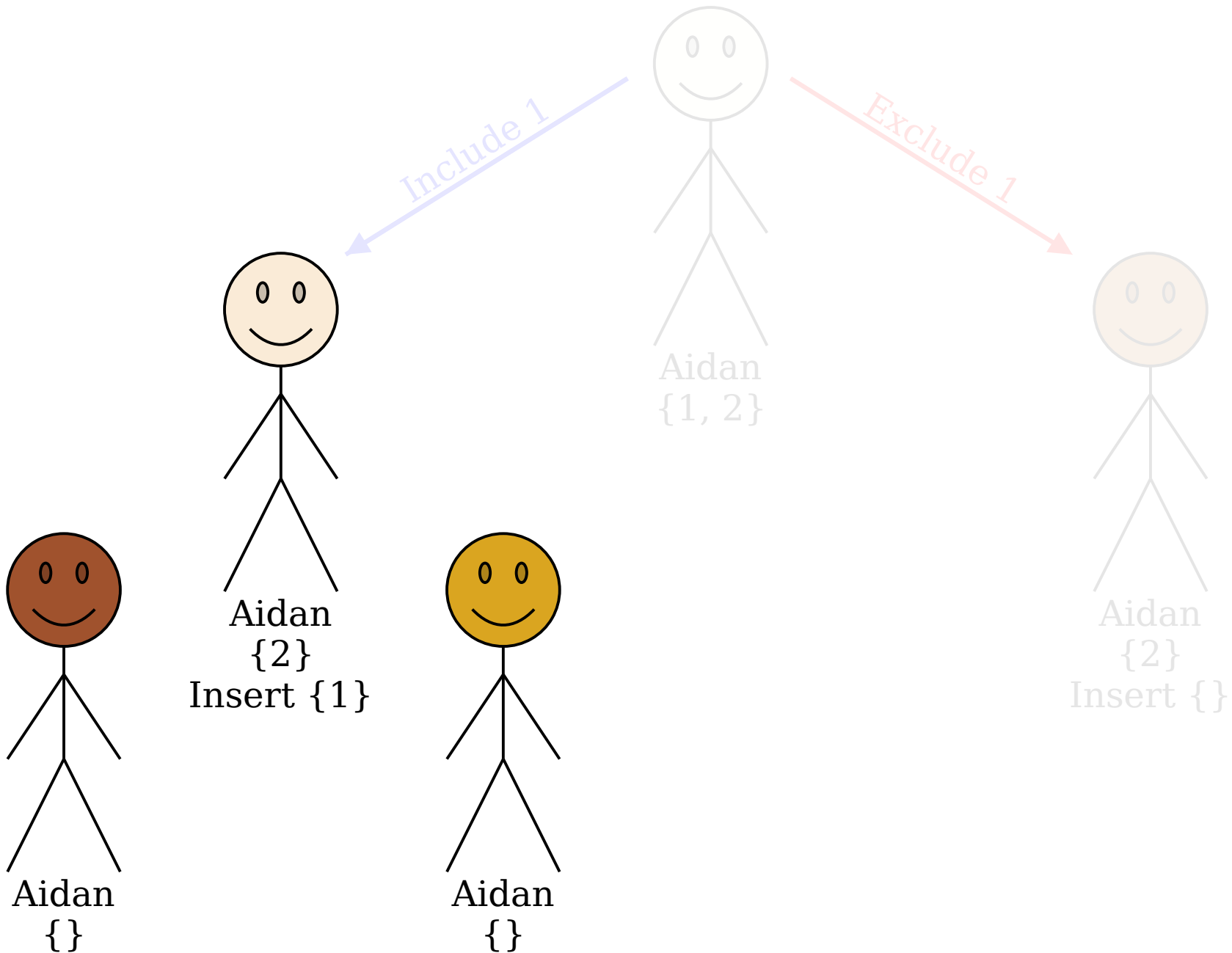
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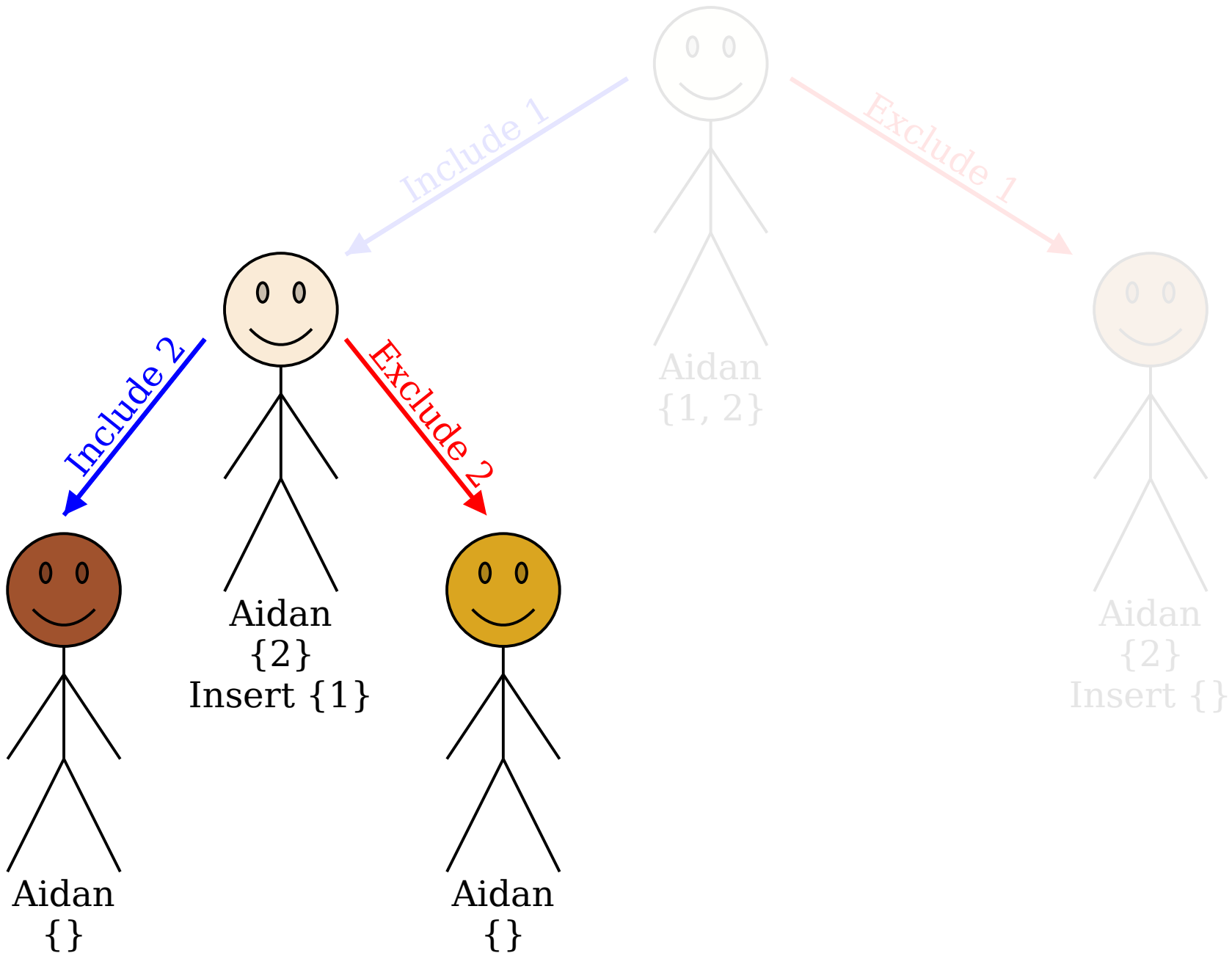
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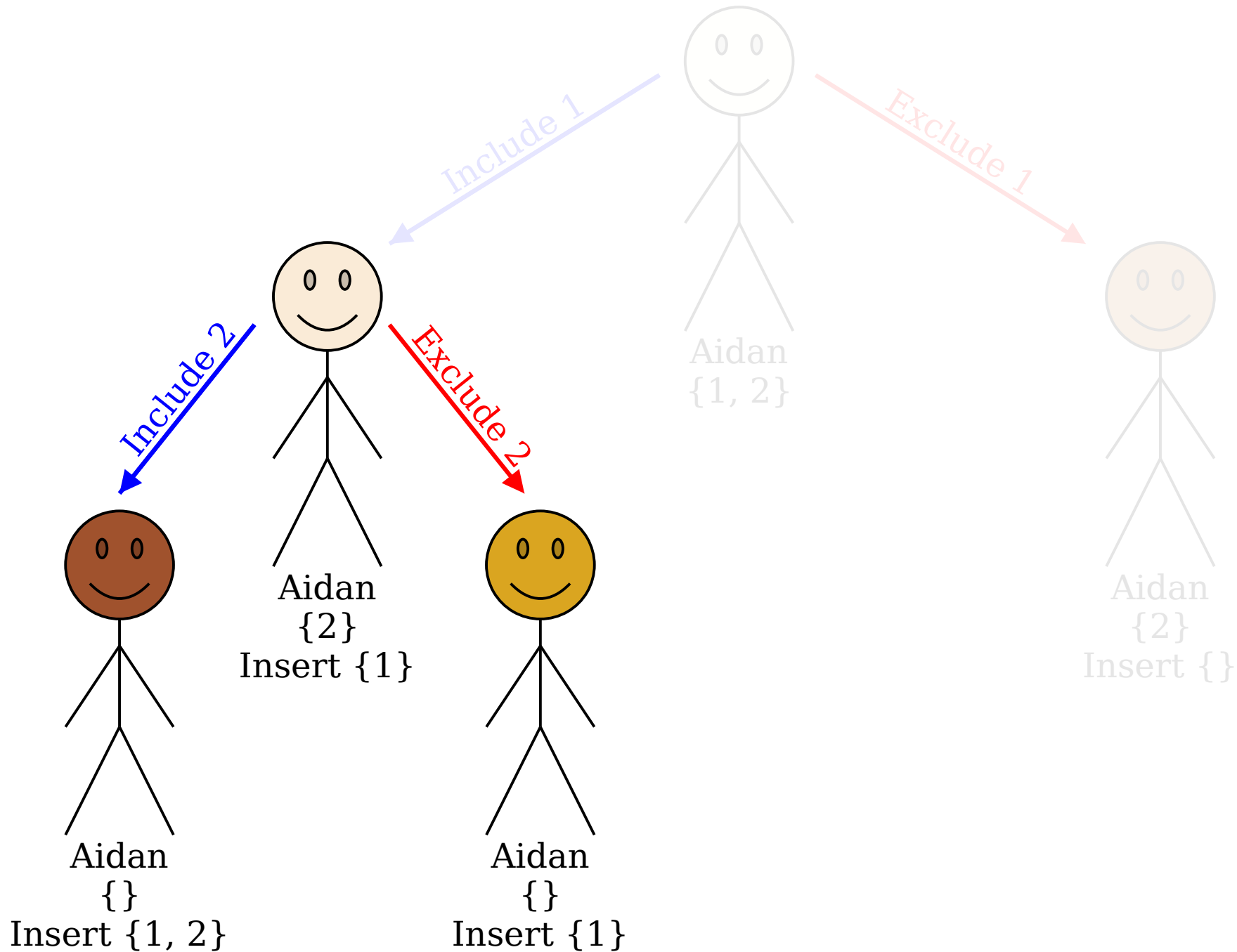
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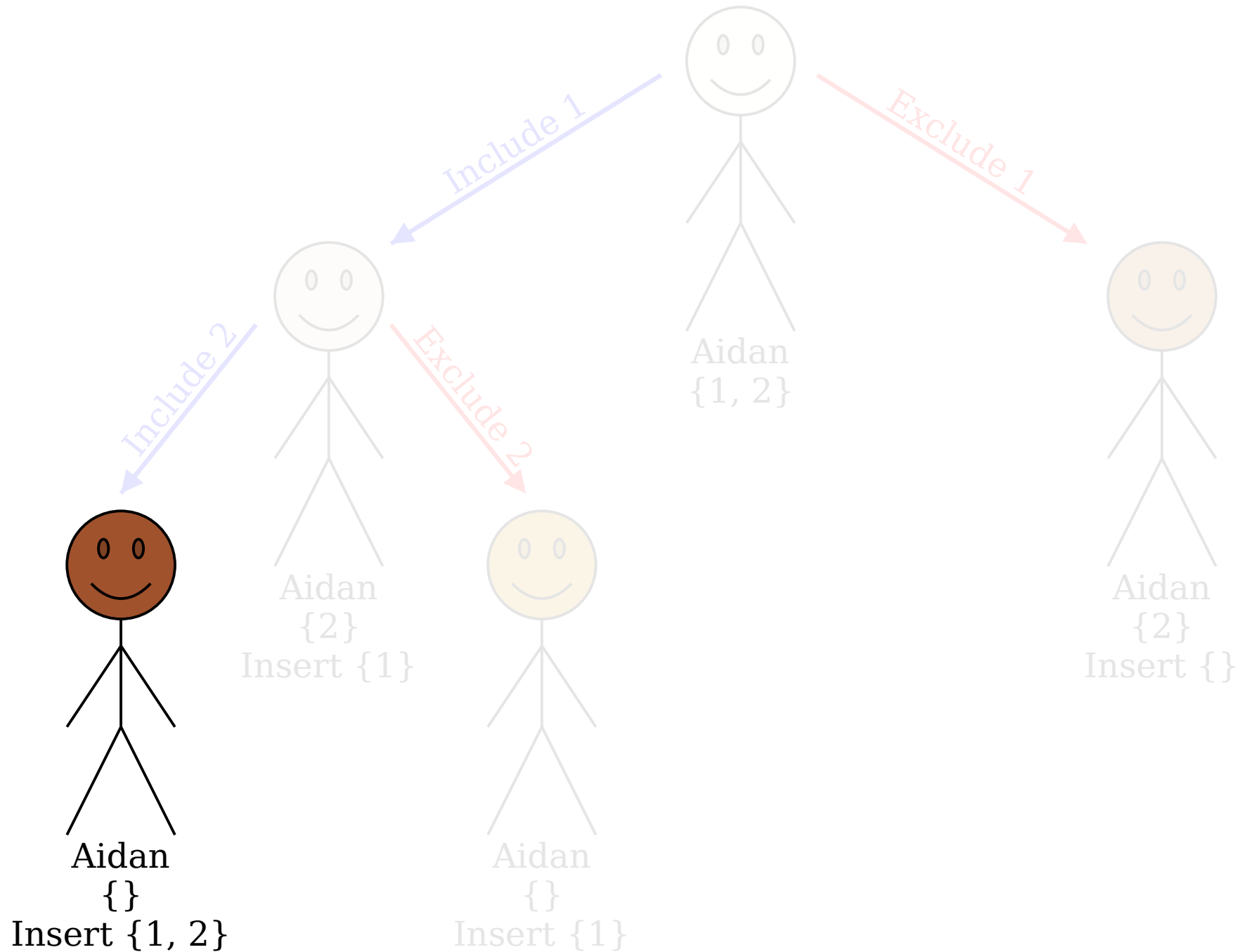
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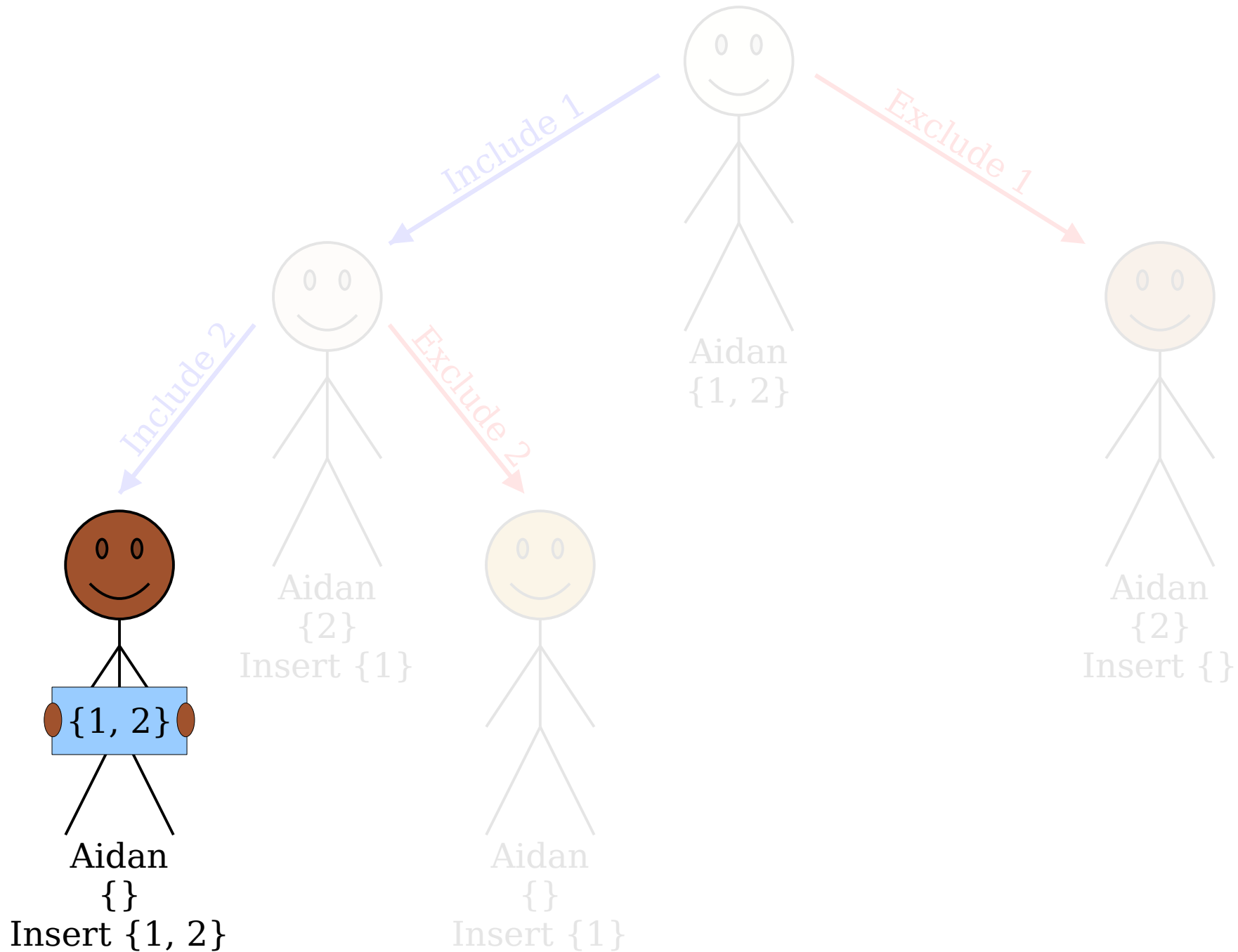
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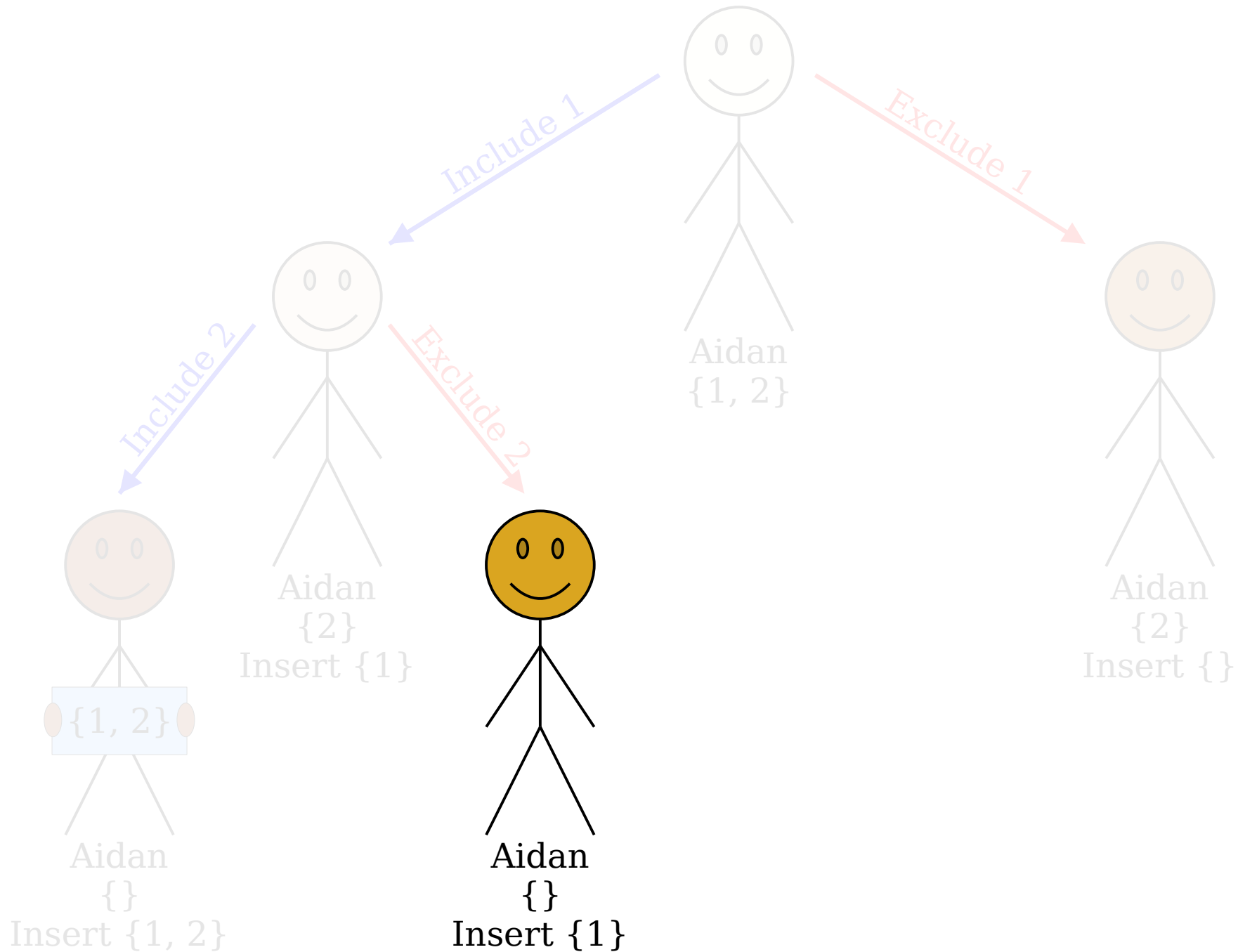
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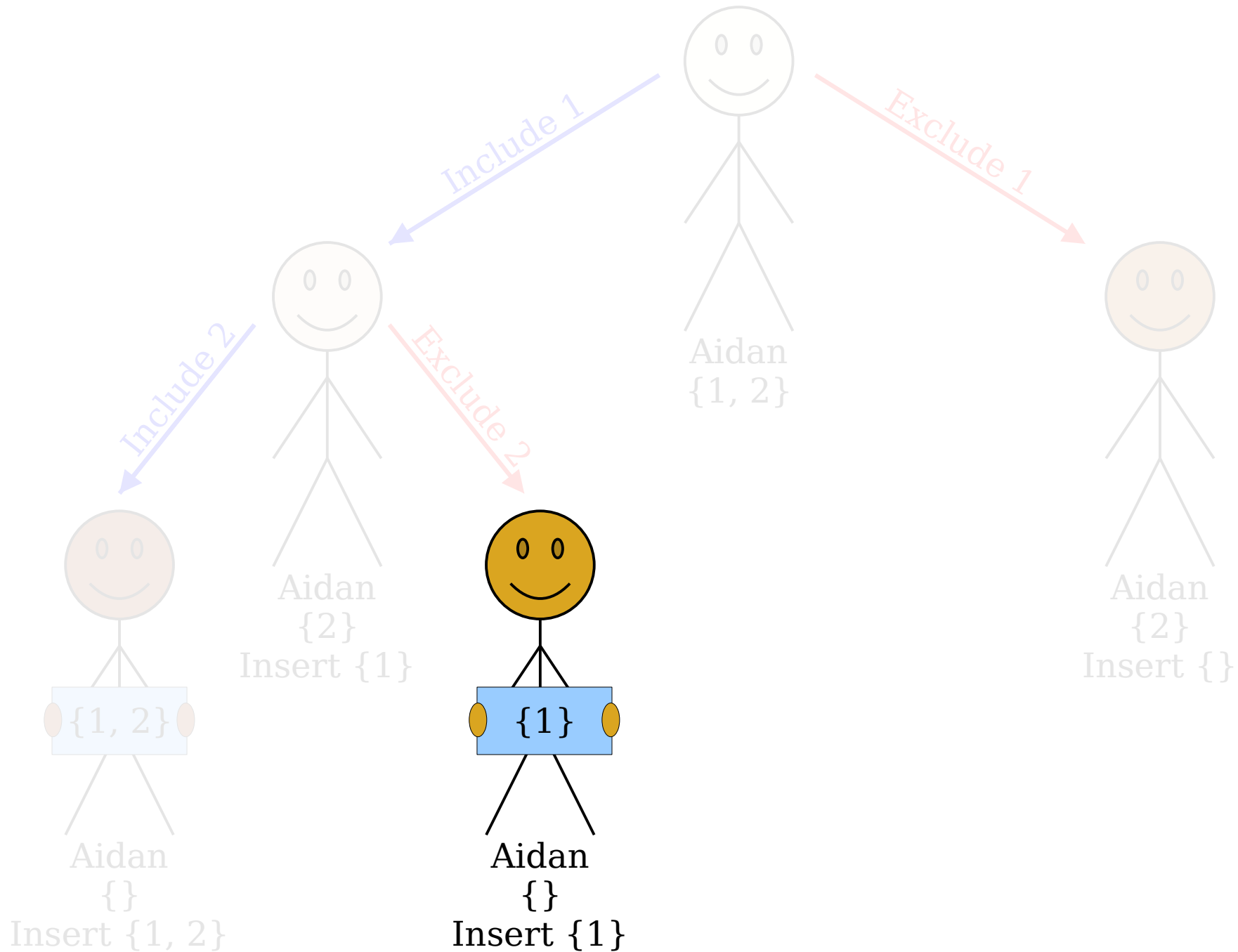
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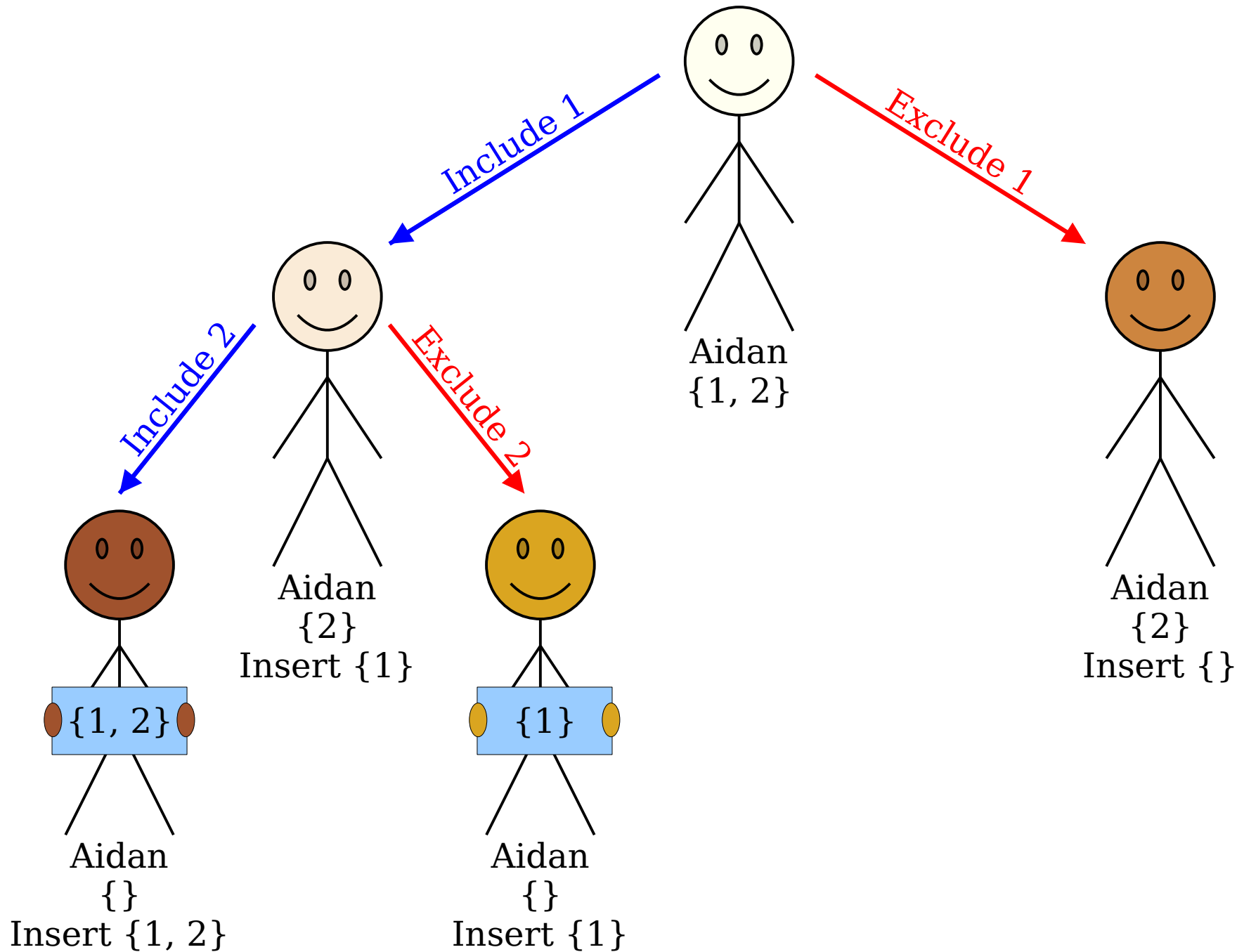
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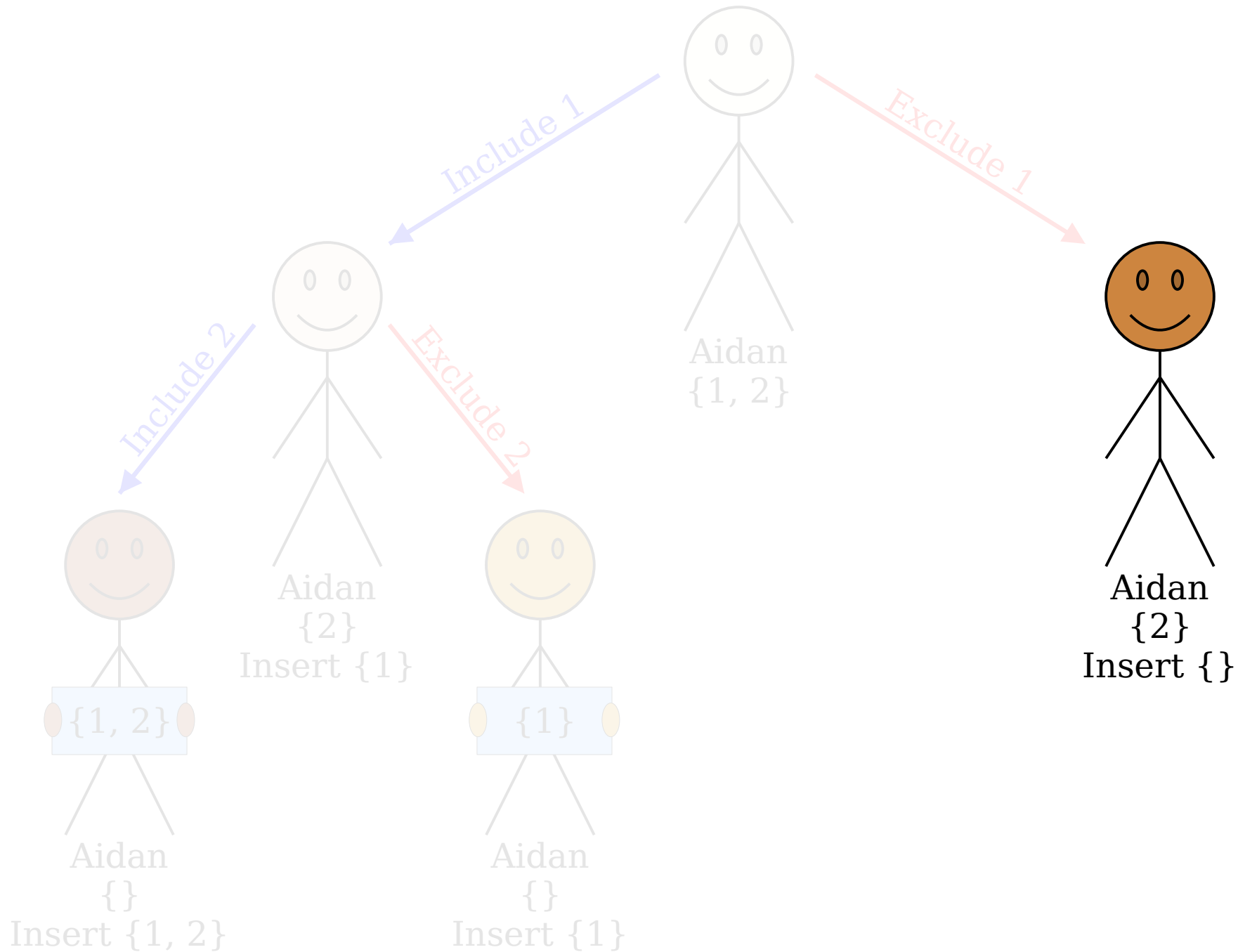
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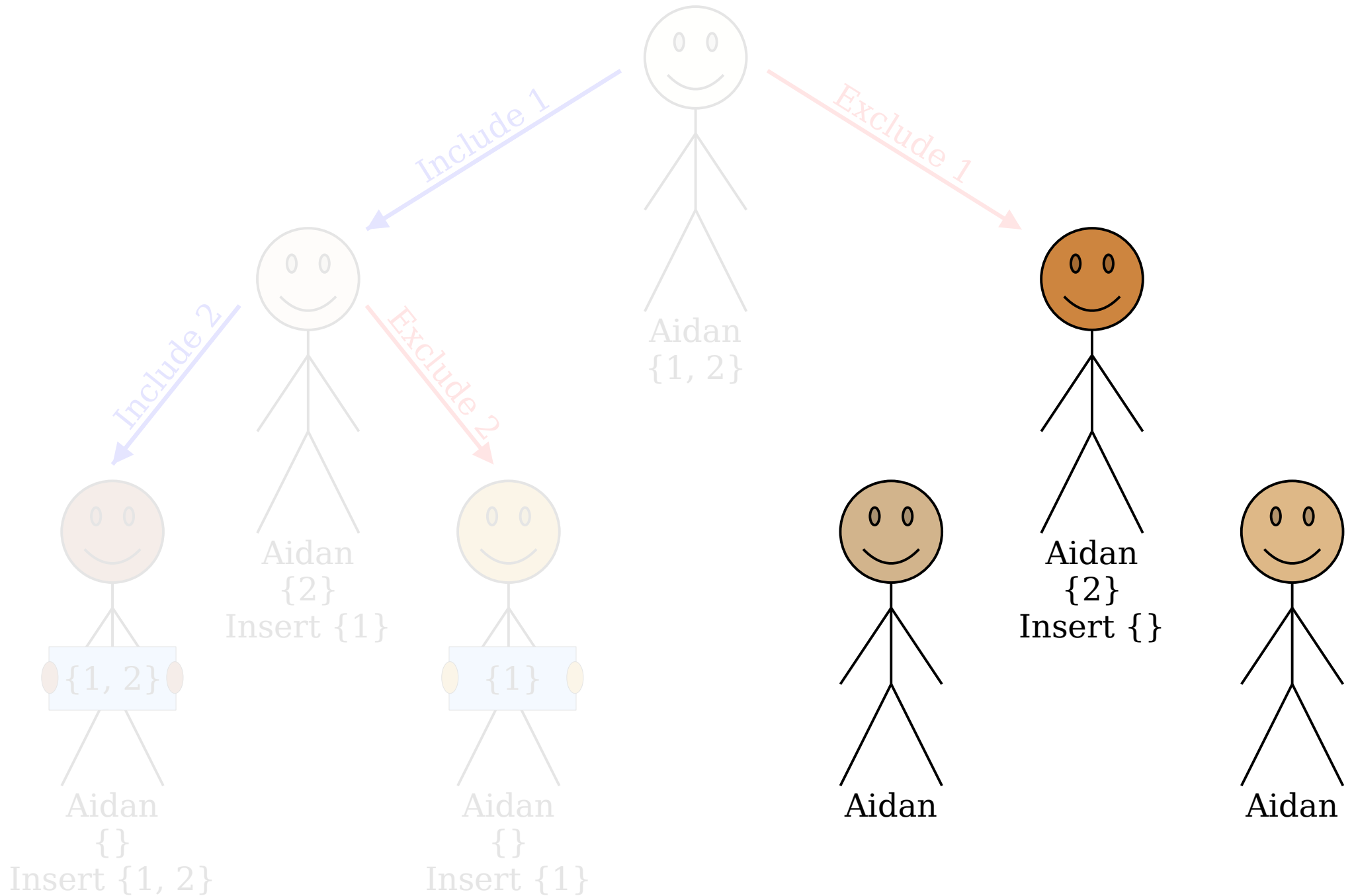
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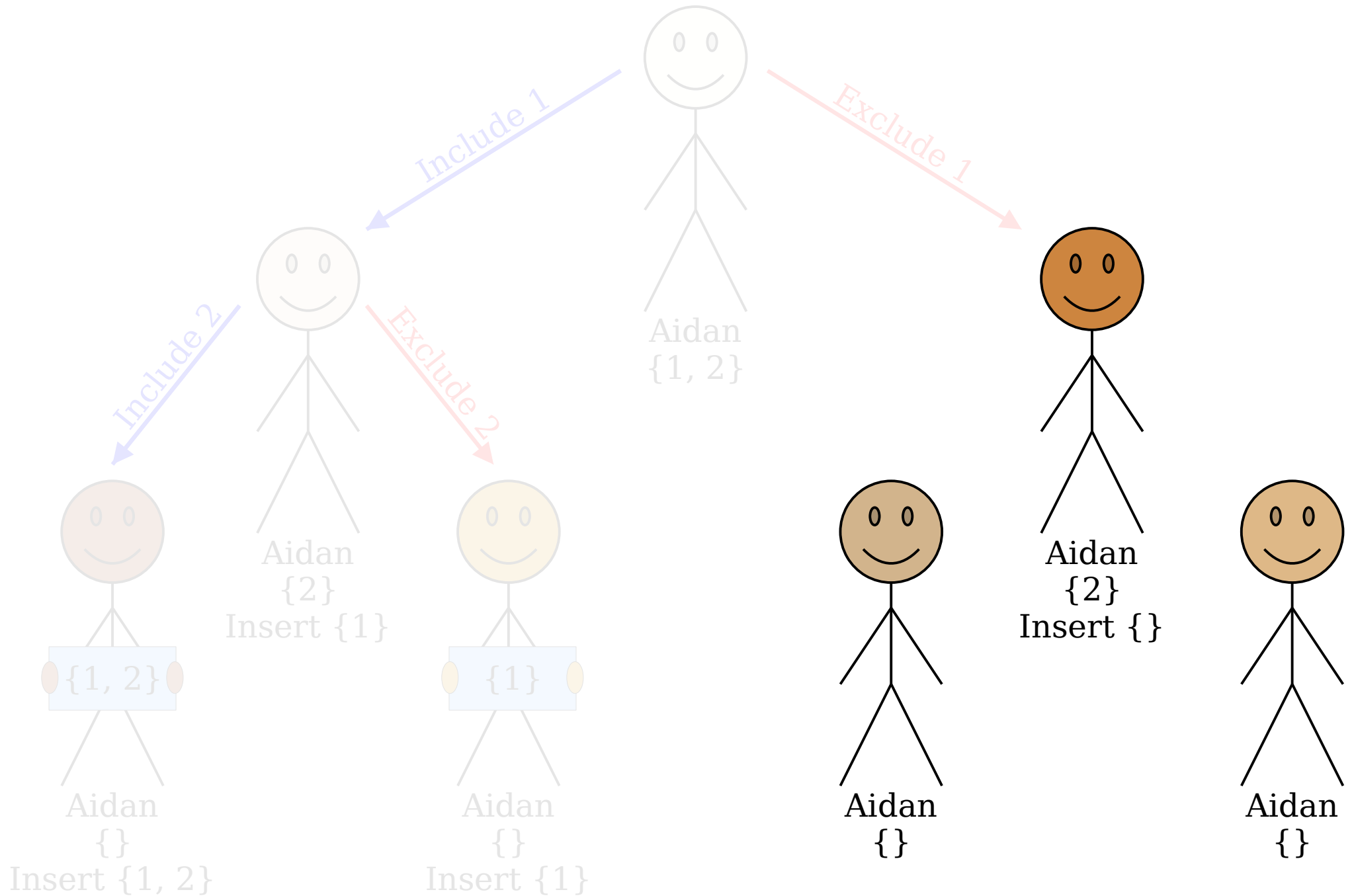
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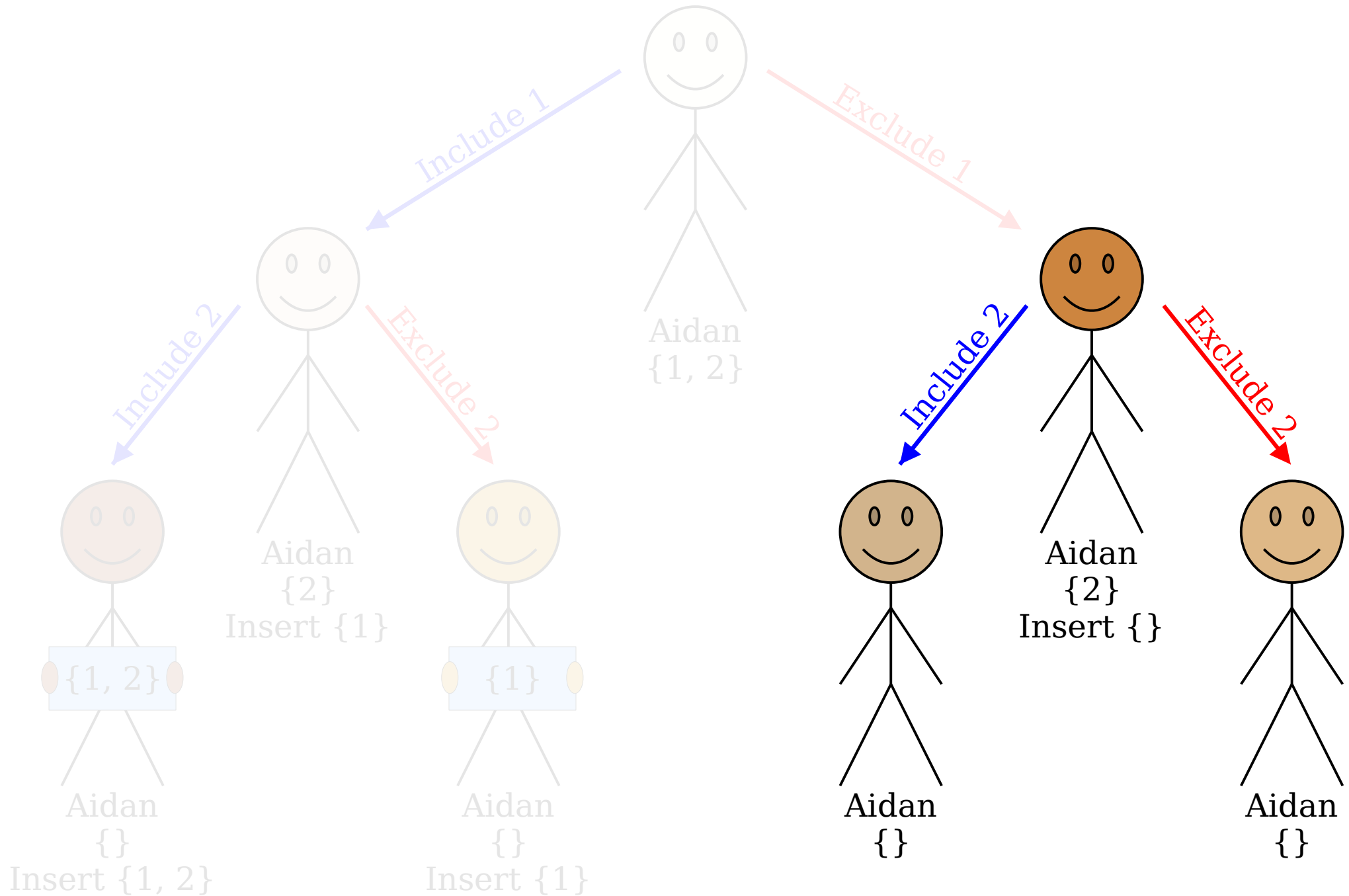
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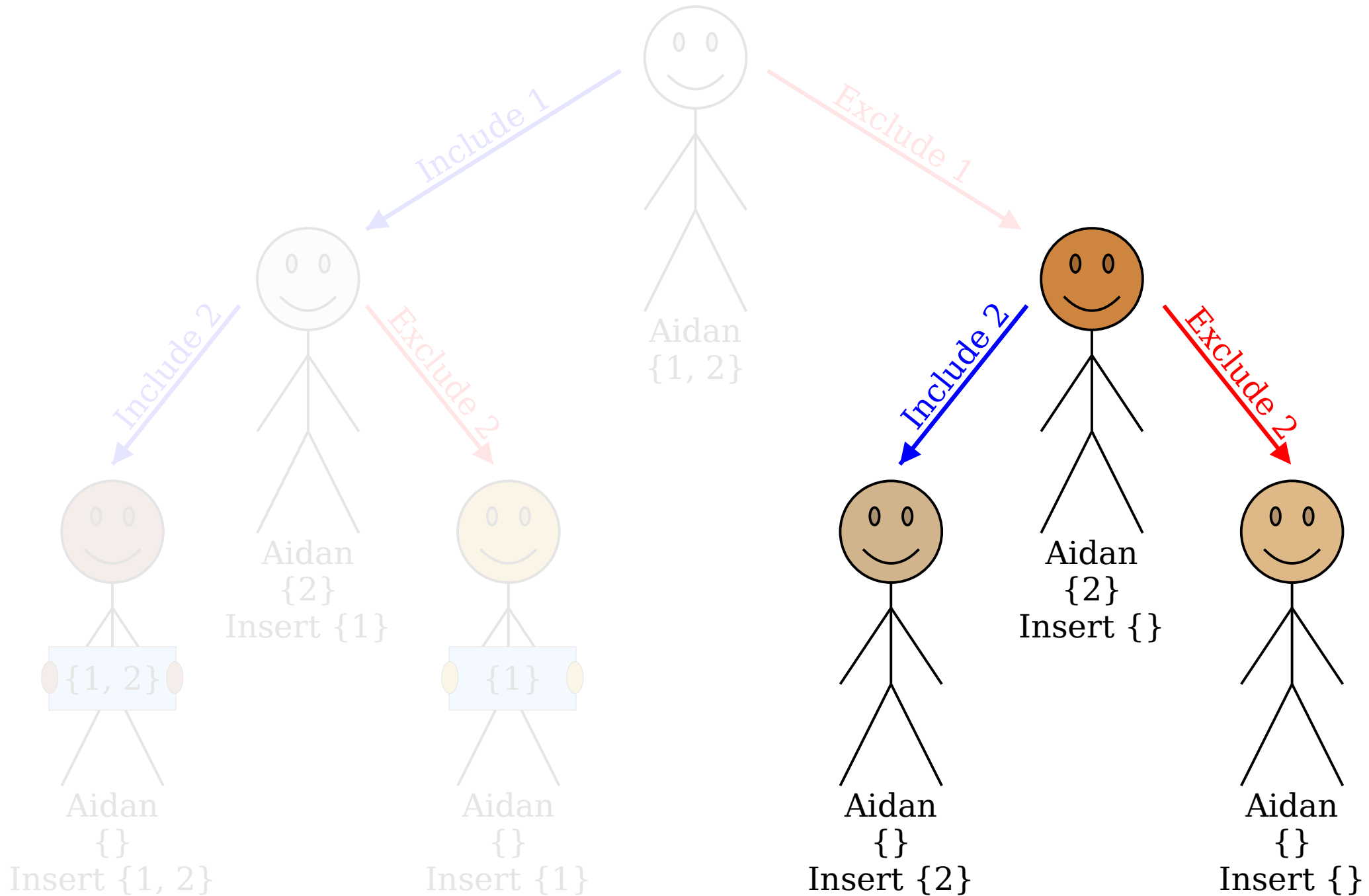
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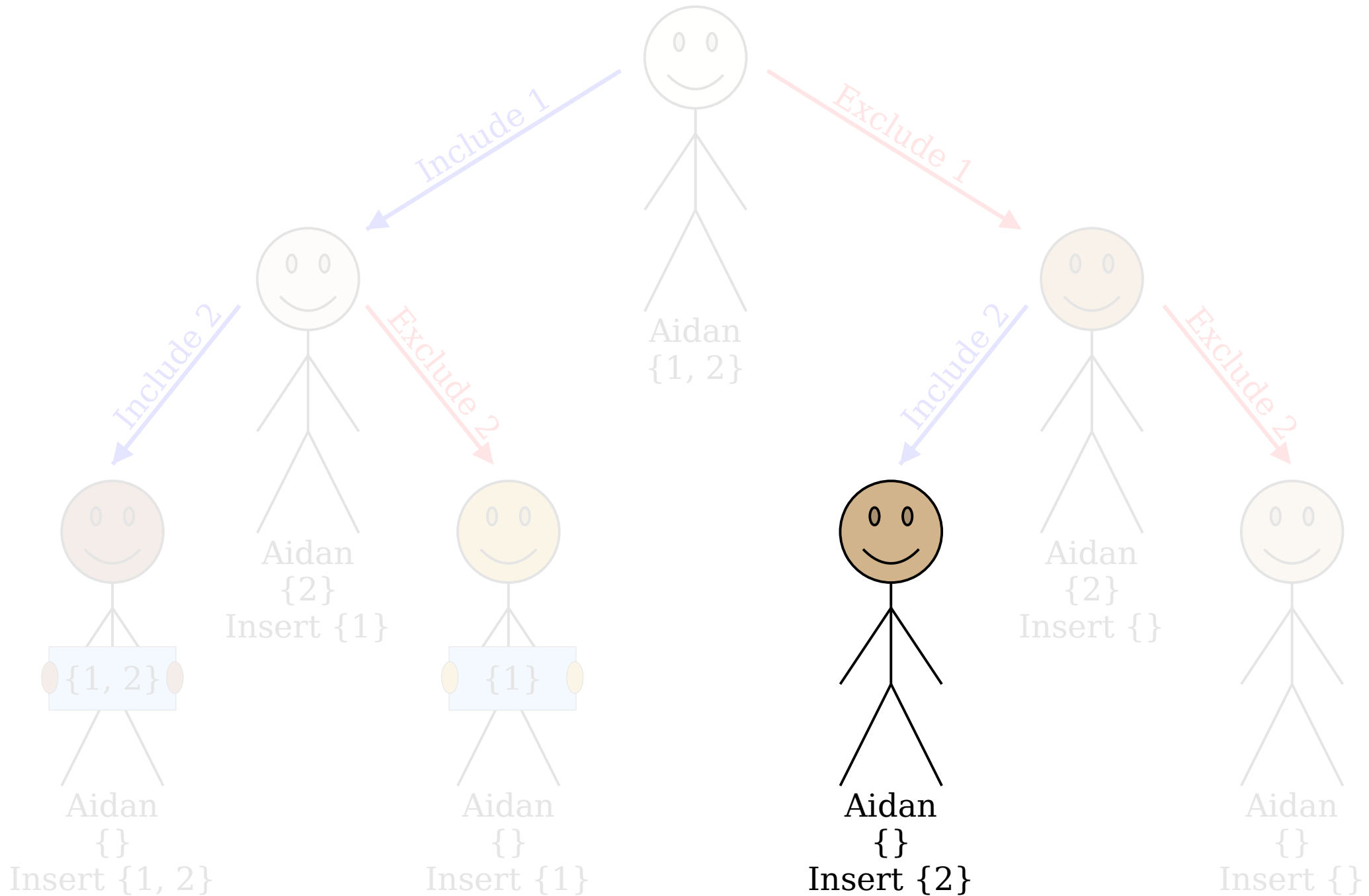
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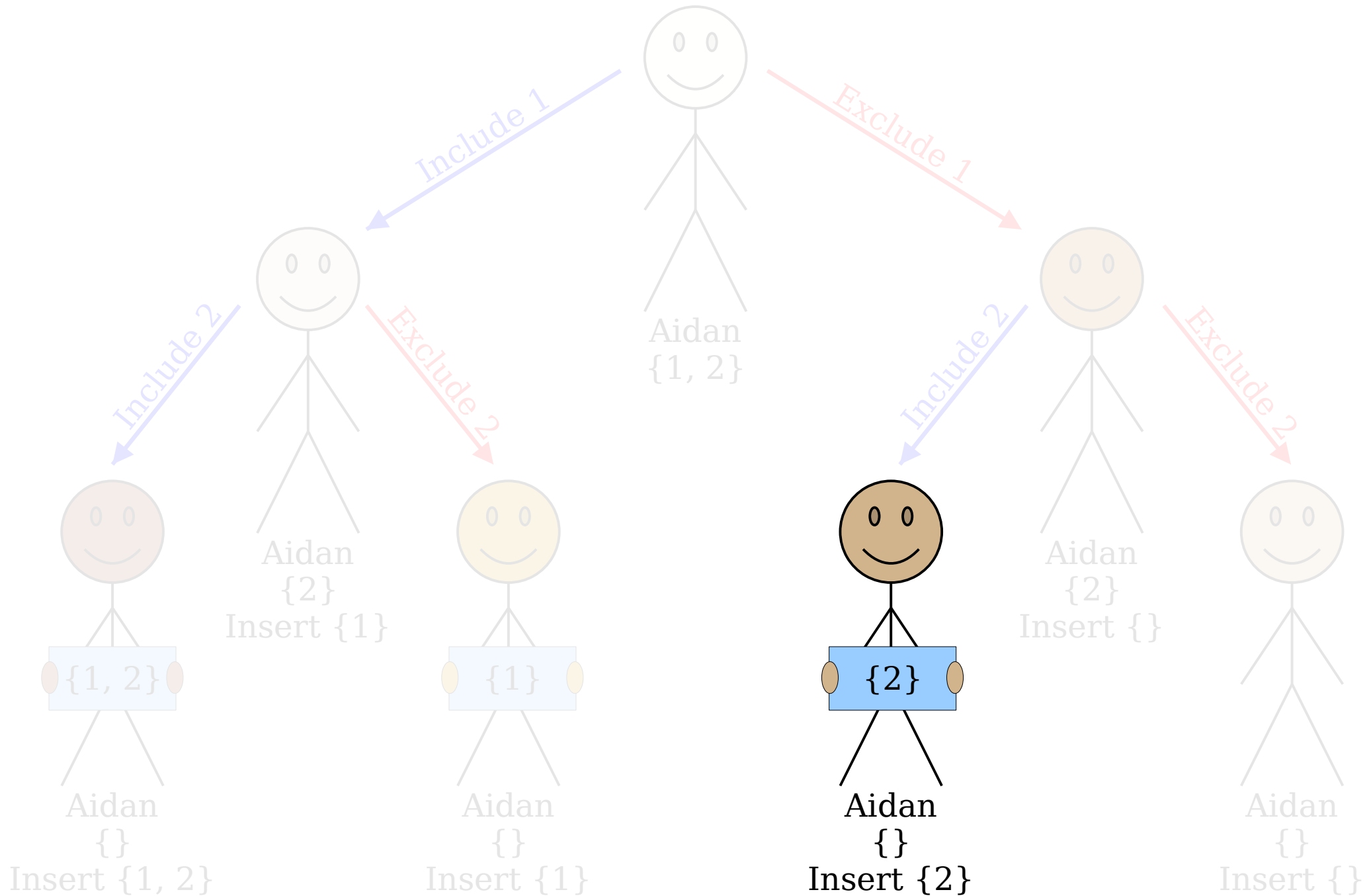
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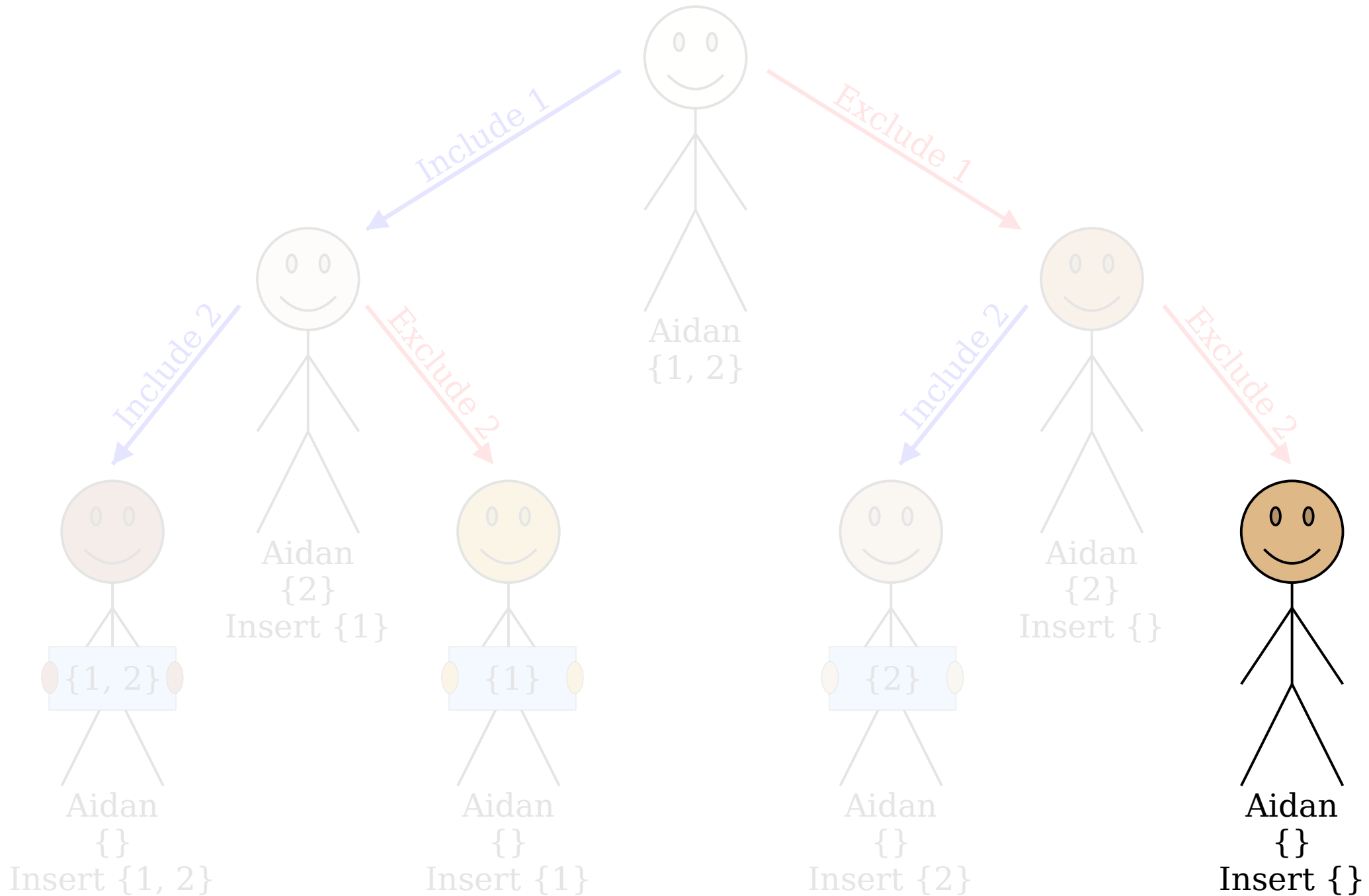
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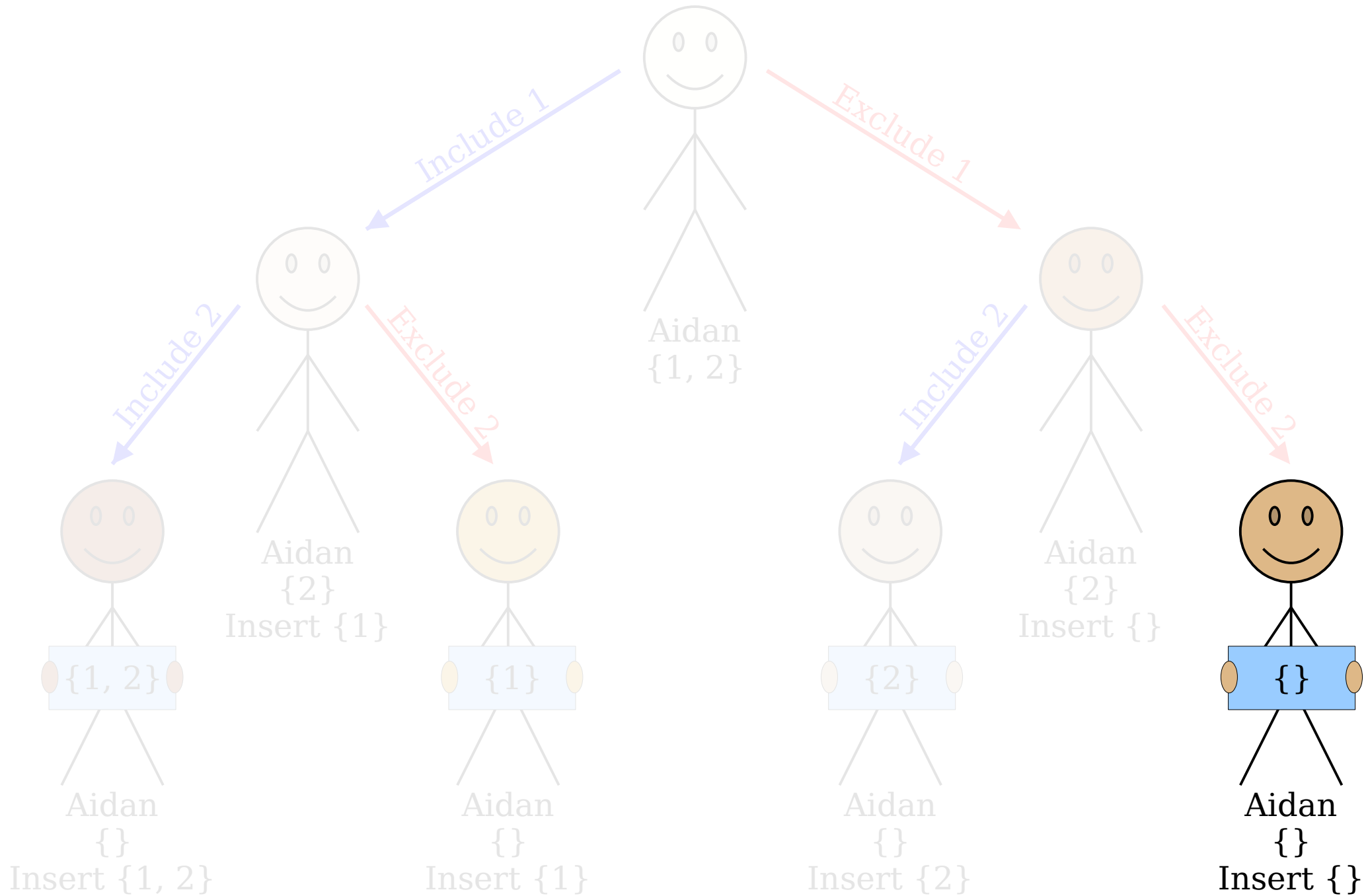
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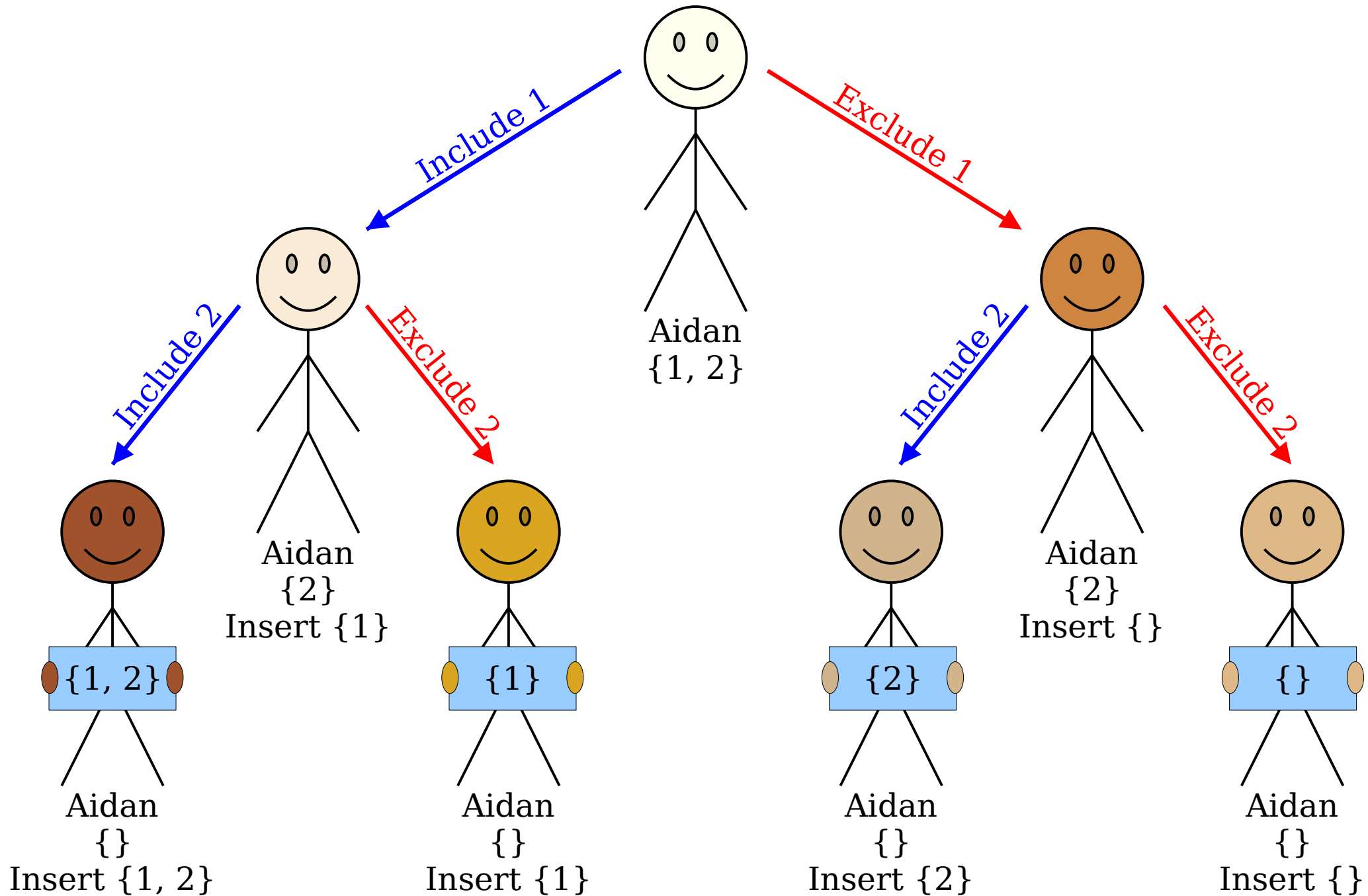
Aidans List Subsets



Aidans List Subsets

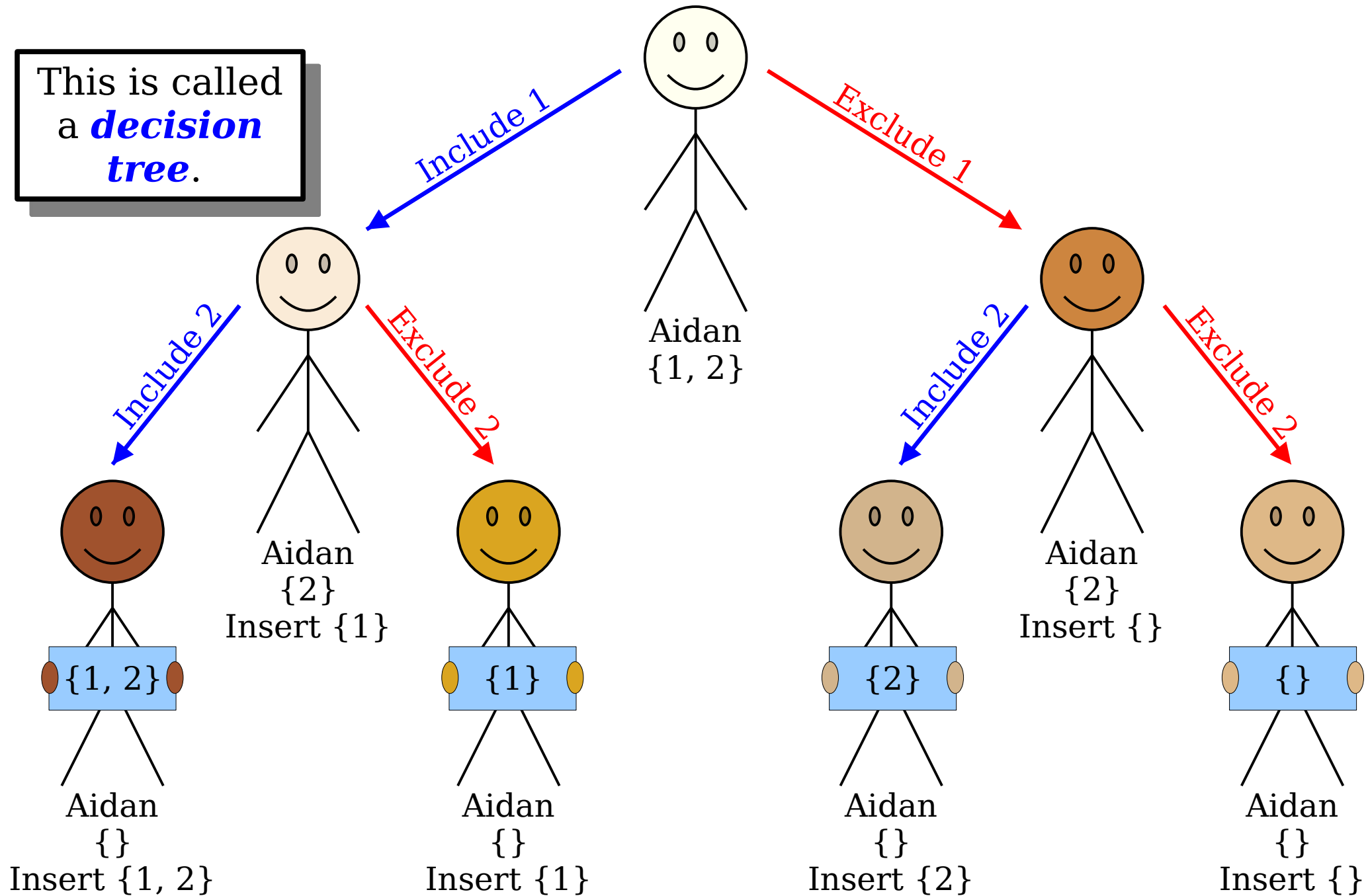


Aidans List Subsets

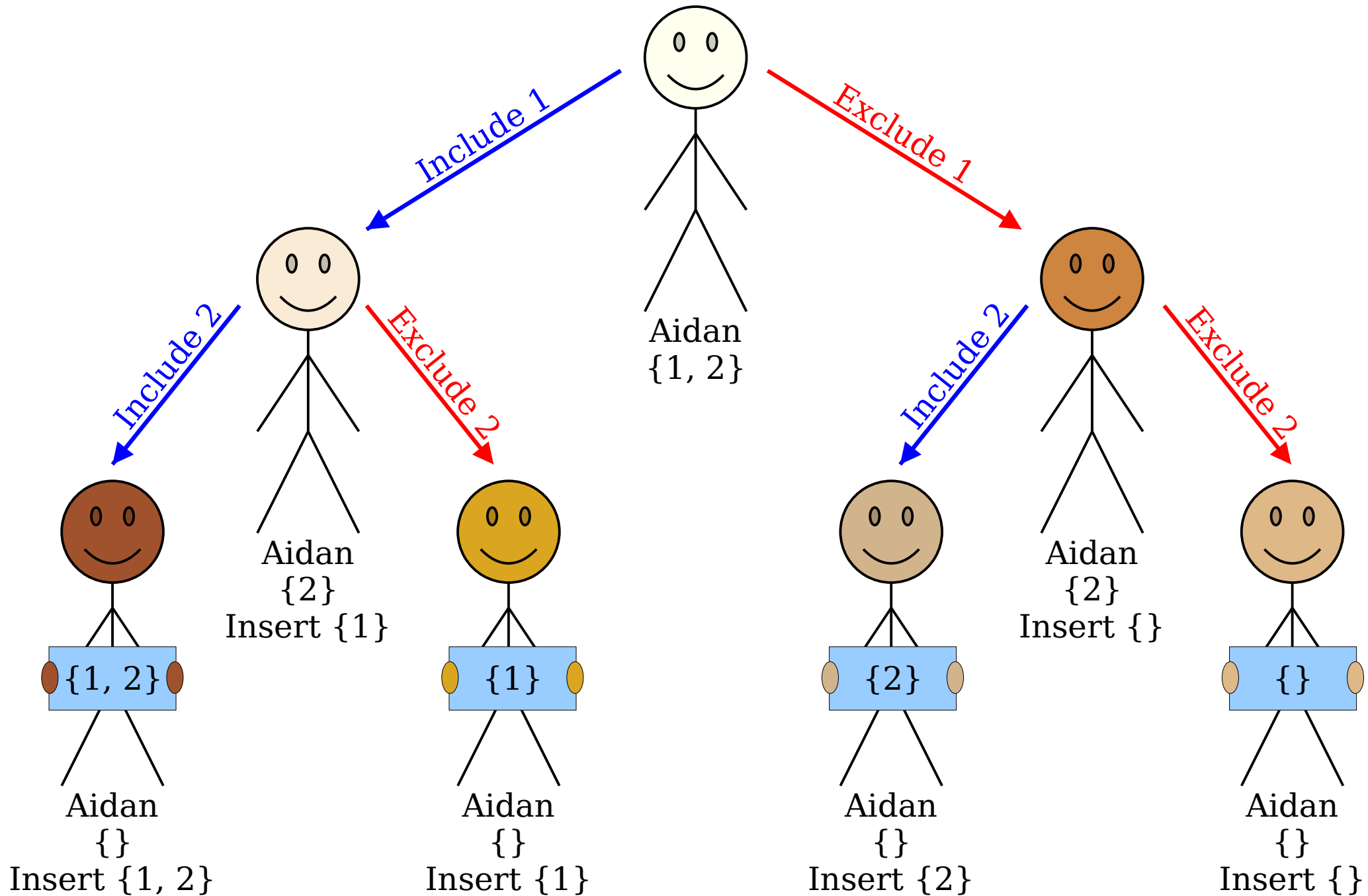


Aidans List Subsets

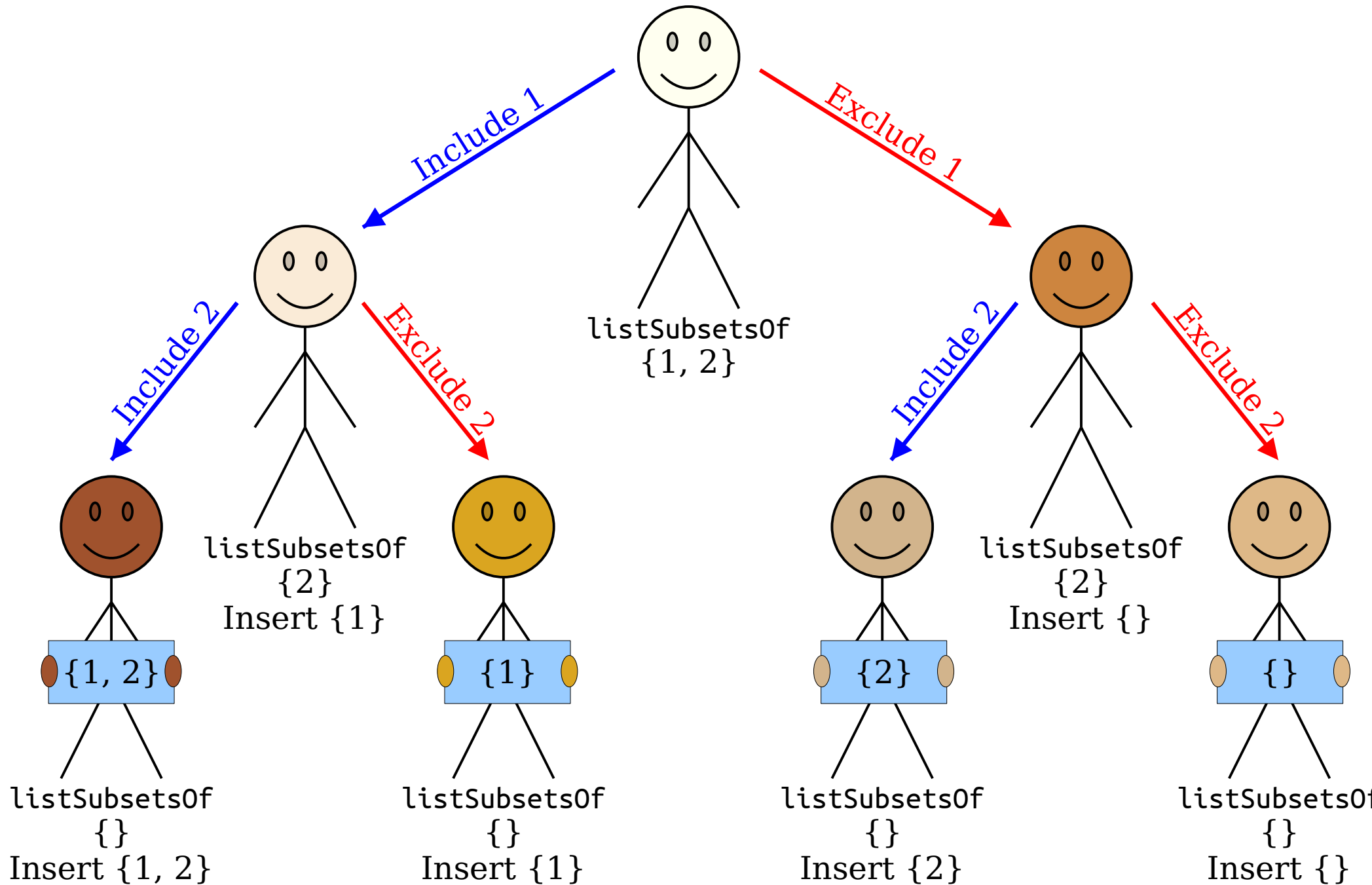
This is called
a **decision tree**.



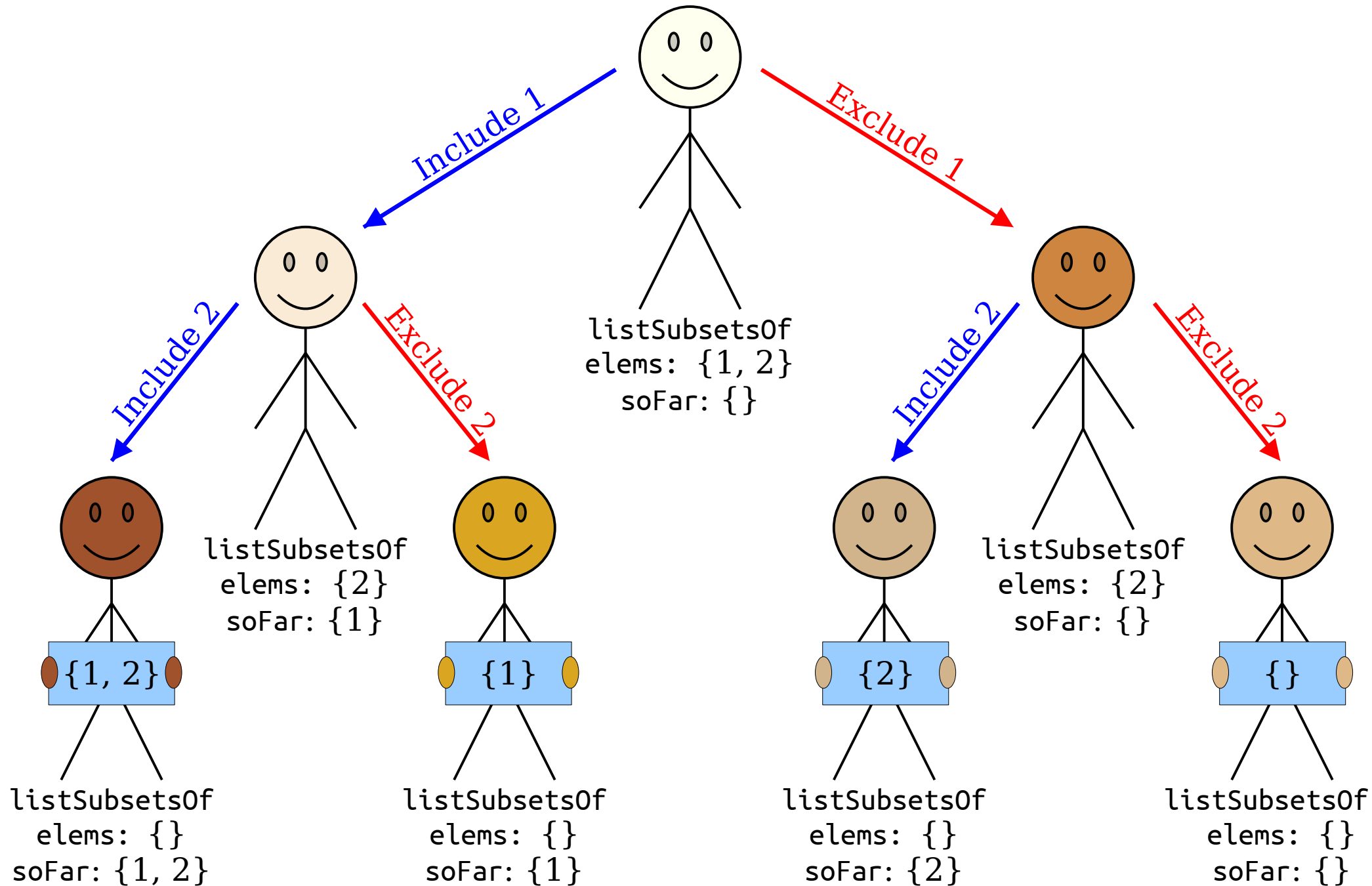
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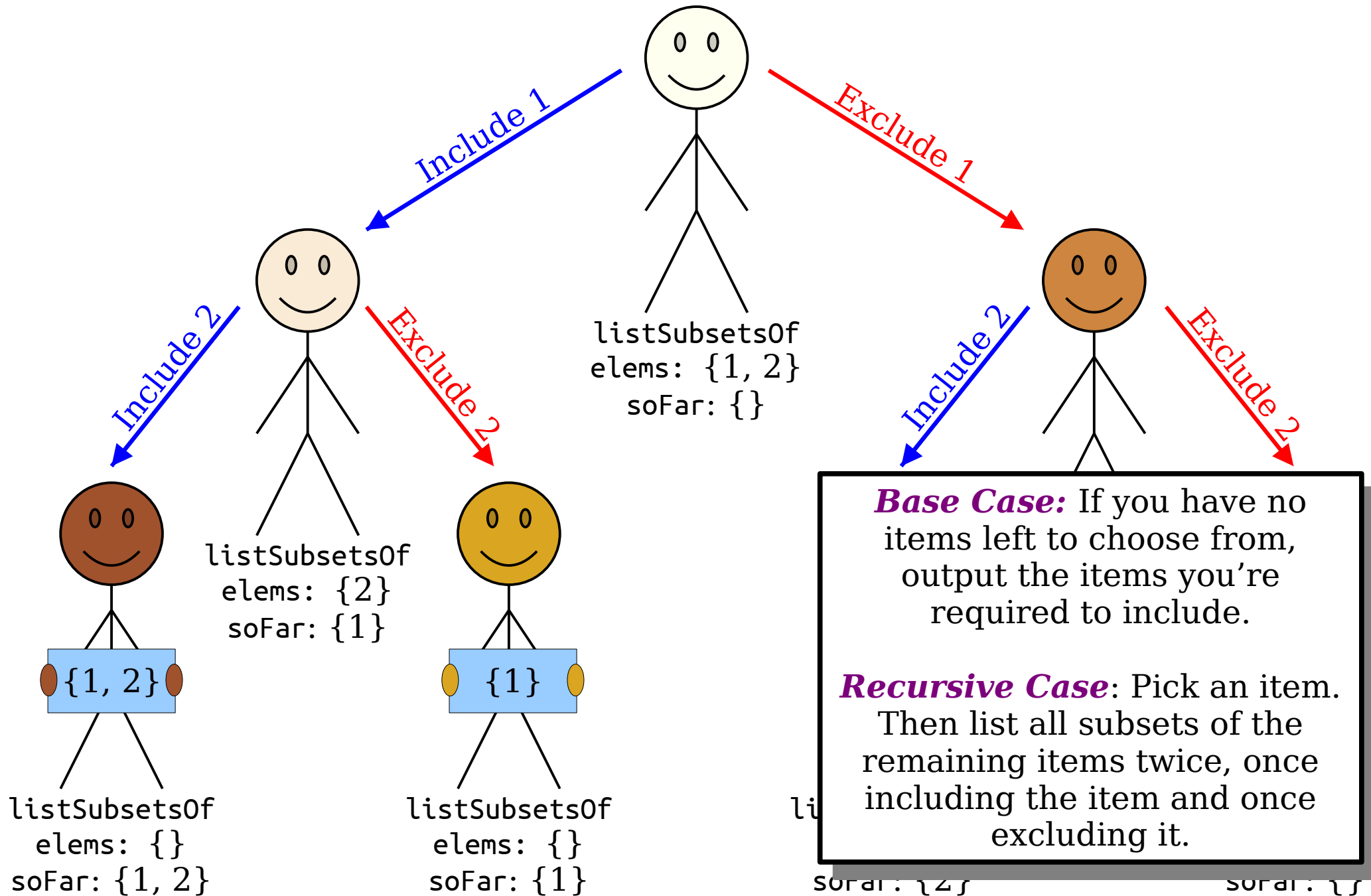
Aidans List Subsets



Aidans List Subsets



Aidans List Subsets



Summary For Today

- Making the ***recursive leap of faith*** and trusting that your recursive calls will perform as expected helps simplify writing recursive code.
- A ***decision tree*** models all the ways you can make choices to arrive at a set of results.

Your Action Items

- ***Read Chapter 8.***
 - There's a lot of great information there about recursive problem-solving, and it's a great resource.
- ***Read the Slide Appendix***
 - There's a trace through how this function works; review this before next lecture.
- ***Finish Assignment 2***
 - If you're following our suggested timetable, at this point you'll have finished Rosetta Stone and will have started working on Rising Tides.
 - Come to LaIR or ask on EdStem if you have any questions!

Next Time

- ***Iteration + Recursion***
 - Combining two techniques together.
- ***Enumerating Permutations***
 - What order should we perform tasks in?

Appendix: Tracing the Recursion

```
int main() {  
    listSubsetsOf({ 1, 2 }, { });  
    return 0;  
}
```

```
int main() {
```

```
    listSubsetsOf({ 1, 2 }, { });
```

```
    return 0;
```

```
}
```

```
int main() {
```

```
void listSubsetsOf(const Set<int>& elems, { { 1, 2 } { }  
               const Set<int>& soFar) {      elems      soFar
```

```
    if (elems.isEmpty()) {  
        cout << soFar << endl;
```

```
    } else {  
        int elem = elems.first();  
        Set<int> remaining = elems - elem;  
  
        /* Option 1: Include this element. */  
        listSubsetsOf(remaining, soFar + elem);  
  
        /* Option 2: Exclude this element. */  
        listSubsetsOf(remaining, soFar);
```

```
    }
```

```
}
```

```
int main() {
```

```
void listSubsetsOf(const Set<int>& elems,  
                  const Set<int>& soFar) {
```

{ 1, 2 }

elems

{ }

soFar

```
    if (elems.isEmpty()) {
```

```
        cout << soFar << endl;
```

```
    } else {
```

```
        int elem = elems.first();
```

```
        Set<int> remaining = elems - elem;
```

```
        /* Option 1: Include this element. */
```

```
        listSubsetsOf(remaining, soFar + elem);
```

```
        /* Option 2: Exclude this element. */
```

```
        listSubsetsOf(remaining, soFar);
```

```
    }
```

```
}
```

```
int main() {
```

```
void listSubsetsOf(const Set<int>& elems, { { 1, 2 } { }  
               const Set<int>& soFar) {      elems      soFar
```

```
    if (elems.isEmpty()) {  
        cout << soFar << endl;
```

```
    } else {
```

```
        int elem = elems.first();  
        Set<int> remaining = elems - elem;
```

```
        /* Option 1: Include this element. */  
        listSubsetsOf(remaining, soFar + elem);
```

```
        /* Option 2: Exclude this element. */  
        listSubsetsOf(remaining, soFar);
```

```
    }
```

```
}
```

```
int main() {
```

```
void listSubsetsOf(const Set<int>& elems,           { { 1, 2 } }  
                  const Set<int>& soFar) {           { { } }  
                                                    elems  soFar
```

```
    if (elems.isEmpty()) {  
        cout << soFar << endl;
```

```
    } else {
```

```
        int elem = elems.first();
```

```
        Set<int> remaining = elems - elem;
```

```
        /* Option 1: Include this element. */
```

```
        listSubsetsOf(remaining, soFar + elem);
```

```
        /* Option 2: Exclude this element. */
```

```
        listSubsetsOf(remaining, soFar);
```

```
    }
```

```
}
```

```
int main() {
```

```
void listSubsetsOf(const Set<int>& elems,  
                  const Set<int>& soFar) {
```

{ 1, 2 }

elems

{ }

soFar

```
if (elems.isEmpty()) {  
    cout << soFar << endl;
```

```
} else {
```

```
    int elem = elems.first();
```

1

elem

```
    Set<int> remaining = elems - elem;
```

```
    /* Option 1: Include this element. */
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```
    listSubsetsOf(remaining, soFar + elem);
```

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    /* Option 2: Exclude this element. */
```

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    listSubsetsOf(remaining, soFar);
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```
}
```

```
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```
int main() {
```

```
void listSubsetsOf(const Set<int>& elems,  
                  const Set<int>& soFar) {
```

{ 1, 2 }

elems

{ }

soFar

```
    if (elems.isEmpty()) {  
        cout << soFar << endl;
```

```
    } else {
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elem

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```

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int main() {
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{ 1, 2 }
elems

{ }
soFar

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    cout << soFar << endl;
```

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} else {
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```
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1
elem

{ 2 }
remaining

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/* Option 1: Include this element. */  
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```

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```

```
Set<int> remaining = elems - elem;
```

2

elem

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elem

{ }

remaining

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int main() {
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```
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}
```

```
}
```

```
}
```

```
{ 1, 2 }
```

```
{ }
```

```
{ 2 }
```

```
{ 1 }
```

```
{ }
```

```
{ 1, 2 }
```

elems

soFar

```
int main() {
```

```
void listSubsetsOf(const Set<int>& elems, const Set<int>& soFar) {
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```

```
    }
```

```
}
```

```
}
```

```
}
```

```
}
```

```
{ 1, 2 }
```

```
{ }
```

```
{ 2 }
```

```
{ 1 }
```

```
{ }
```

```
{ 1, 2 }
```

elems

soFar

```
int main() {
```

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void listSubsetsOf(const Set<int>& elems, { 1, 2 } { }
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Program

```
int main() {
```

```
void listSubsetsOf(const Set<int>& elems, { 1, 2 } { }
```

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void listSubsetsOf(const Set<int>& elems, { } { 1, 2 }  
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        listSubsetsOf(remaining, soFar);
```

```
    }
```

```
}
```



Program

{1, 2}

```
int main() {
```

```
void listSubsetsOf(const Set<int>& elems, { 1, 2 } { }
```

```
void listSubsetsOf(const Set<int>& elems, { 2 } { 1 }
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```
void listSubsetsOf(const Set<int>& elems, { } { 1, 2 }  
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        cout << soFar << endl;
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```
}
```



Program

{1, 2}

```
int main() {
```

```
void listSubsetsOf(const Set<int>& elems, { 1, 2 } { }
```

```
void listSubsetsOf(const Set<int>& elems, { { 2 } { 1 }  
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```
if (elems.isEmpty()) {  
    cout << soFar << endl;
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```
} else {
```

```
    int elem = elems.first();
```

```
    Set<int> remaining = elems - elem;
```

2

elem

{ }

remaining

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Program

{1, 2}

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Program

{1, 2}

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Program

{1, 2}

```
int main() {
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```
void listSubsetsOf(const Set<int>& elems, const Set<int>& soFar) {
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Program

{1, 2}

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Program

{1, 2}

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Program

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{1, 2}
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```
{1}
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Program

```
{1, 2}  
{1}
```

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elem

{ }

remaining

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Program

{1, 2}

{1}

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}
```

```
}
```



Program

{1, 2}

{1}

```
int main() {
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void listSubsetsOf(const Set<int>& elems,  
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{ 1, 2 }
elems

{ }
soFar

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        int elem = elems.first();  
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```

1
elem

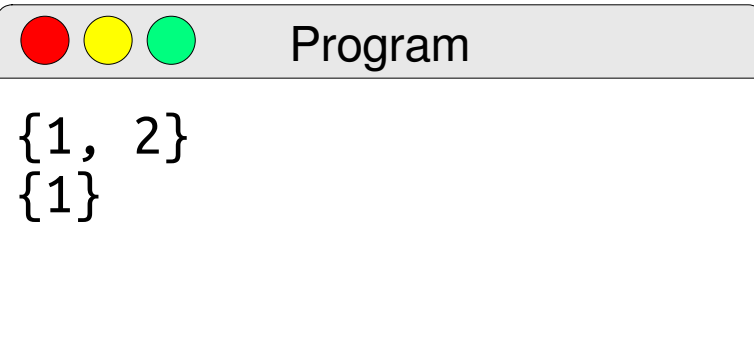
{ 2 }

```
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1
elem

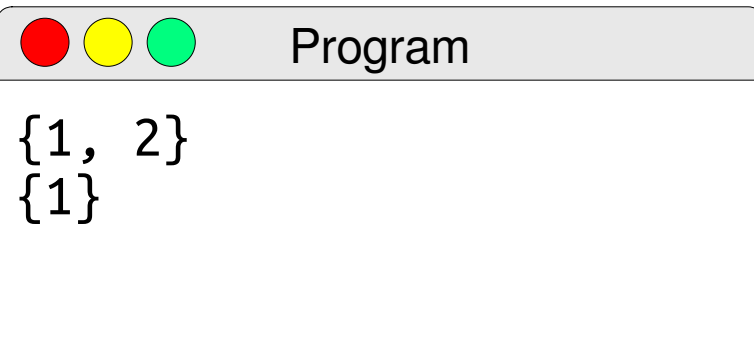
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```
int main() {
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```
void listSubsetsOf(const Set<int>& elems, { 1, 2 } { }
```

```
void listSubsetsOf(const Set<int>& elems, { 2 } { }  
                  const Set<int>& soFar) { elems soFar
```

```
    if (elems.isEmpty()) {  
        cout << soFar << endl;  
    } else {  
        int elem = elems.first();  
        Set<int> remaining = elems - elem;  
  
        /* Option 1: Include this element. */  
        listSubsetsOf(remaining, soFar + elem);  
  
        /* Option 2: Exclude this element. */  
        listSubsetsOf(remaining, soFar);  
    }  
}
```



Program

```
{1, 2}  
{1}
```

```
int main() {
```

```
void listSubsetsOf(const Set<int>& elems, { 1, 2 } { }
```

```
void listSubsetsOf(const Set<int>& elems, { 2 } { }  
                  const Set<int>& soFar) { elems soFar
```

```
    if (elems.isEmpty()) {
```

```
        cout << soFar << endl;
```

```
    } else {
```

```
        int elem = elems.first();
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```
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        listSubsetsOf(remaining, soFar);
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```
    }
```

```
}
```

Program

```
{1, 2}  
{1}
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```
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```



Program

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{1, 2}  
{1}
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```

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    } else {
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```
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```
        listSubsetsOf(remaining, soFar);
```

```
    }
```

```
}
```

Program

```
{1, 2}  
{1}
```

```
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```

```
void listSubsetsOf(const Set<int>& elems, { 1, 2 } { }
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```
void listSubsetsOf(const Set<int>& elems, { { 2 } { }  
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```

```
    if (elems.isEmpty()) {  
        cout << soFar << endl;
```

```
    } else {
```

```
        int elem = elems.first();
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```
        Set<int> remaining = elems - elem; elem
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Program

```
{1, 2}
```

```
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```

```
    }
```

```
}
```

2

elem



Program

```
{1, 2}  
{1}
```

```
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```

```
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Program

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```
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```

```
}
```



Program

```
{1, 2}
```

```
{1}
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Program

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{1, 2}
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{1}
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```
    }
```

```
}
```



Program

```
{1, 2}
```

```
{1}
```

```
{2}
```

```
int main() {
```

```
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Program

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{1, 2}  
{1}  
{2}
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Program

```
{1, 2}  
{1}  
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```
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```

```
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```

2

{ }

elem

remaining

```
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listSubsetsOf(remaining, soFar);
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Program

```
{1, 2}  
{1}  
{2}
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```
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    }  
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```



Program

```
{1, 2}  
{1}  
{2}
```

```
int main() {
```

```
void listSubsetsOf(const Set<int>& elems, const Set<int>& soFar) {
```

```
void listSubsetsOf(const Set<int>& elems, const Set<int>& soFar) {
```

```
void listSubsetsOf(const Set<int>& elems, const Set<int>& soFar) {
```

```
if (elems.isEmpty()) {
```

```
    cout << soFar << endl;
```

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} else {
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    int elem = elems.first();
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    Set<int> remaining = elems - elem;
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```
}
```

```
}
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Program

```
{1, 2}
```

```
{1}
```

```
{2}
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int main() {
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```

```
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Program

```
{1, 2}
```

```
{1}
```

```
{2}
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Program

```
{1, 2}  
{1}  
{2}  
{}
```

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```



Program

```
{1, 2}  
{1}  
{2}  
{ }
```

```
int main() {
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const Set<int>& soFar) { elems soFar
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```
if (elems.isEmpty()) {  
    cout << soFar << endl;
```

```
} else {
```

```
    int elem = elems.first();
```

```
    Set<int> remaining = elems - elem;
```

2

{ }

elem

remaining

```
/* Option 1: Include this element. */
```

```
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listSubsetsOf(remaining, soFar);
```



Program

```
{1, 2}  
{1}  
{2}  
{ }
```

```
int main() {
```

```
void listSubsetsOf(const Set<int>& elems,
```

```
{ 1, 2 }
```

```
{ }
```

```
void listSubsetsOf(const Set<int>& elems,
```

```
const Set<int>& soFar) {
```

```
{ 2 }
```

elems

```
{ }
```

soFar

```
if (elems.isEmpty()) {  
    cout << soFar << endl;
```

```
} else {
```

```
    int elem = elems.first();
```

```
    Set<int> remaining = elems - elem;
```

```
2
```

elem

```
{ }
```

remaining

```
/* Option 1: Include this element. */
```

```
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```

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```

```
listSubsetsOf(remaining, soFar);
```

```
}
```

```
}
```

```
}
```

```
}
```



Program

```
{1, 2}  
{1}  
{2}  
{ }
```

```
int main() {
```

```
void listSubsetsOf(const Set<int>& elems,  
                  const Set<int>& soFar) {
```

{ 1, 2 }
elems

{ }
soFar

```
    if (elems.isEmpty()) {  
        cout << soFar << endl;
```

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        int elem = elems.first();
```

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        Set<int> remaining = elems - elem;
```

1
elem

{ 2 }

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        listSubsetsOf(remaining, soFar + elem);
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        listSubsetsOf(remaining, soFar);
```

```
    }
```

```
}
```

Program

```
{1, 2}  
{1}  
{2}  
{}
```

```
int main() {
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```
void listSubsetsOf(const Set<int>& elems,  
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{ 1, 2 }

elems

{ }

soFar

```
    if (elems.isEmpty()) {  
        cout << soFar << endl;
```

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```
        int elem = elems.first();
```

```
        Set<int> remaining = elems - elem;
```

1

elem

{ 2 }

```
        /* Option 1: Include this element. */
```

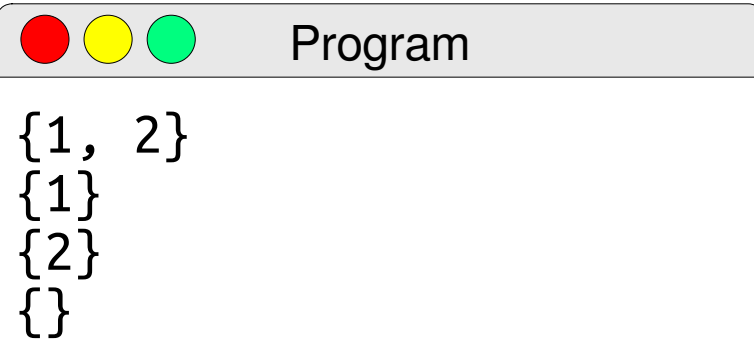
```
        listSubsetsOf(remaining, soFar + elem);
```

```
        /* Option 2: Exclude this element. */
```

```
        listSubsetsOf(remaining, soFar);
```

```
    }
```

```
}
```



```
Program  
{1, 2}  
{1}  
{2}  
{}
```

```
int main() {
```

```
    listSubsetsOf({ 1, 2 }, { });
```

```
    return 0;
```

```
}
```



Program

{1, 2}

{1}

{2}

{}